

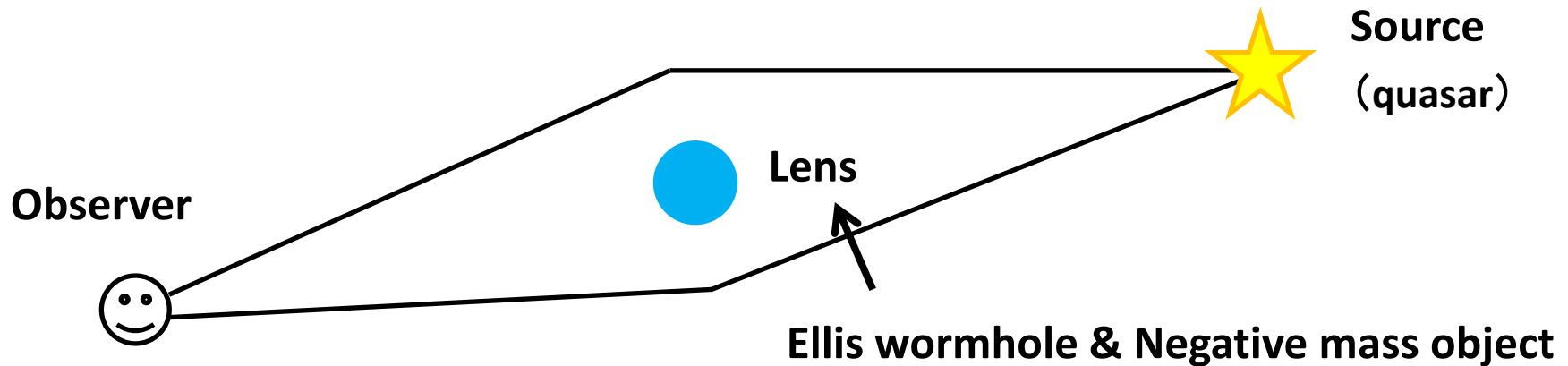
**Observational Upper Bound
on the Cosmic Abundances of
Negative-mass Compact Objects
and Ellis Wormholes
from the SDSS Quasar Lens Search**

**Ryuichi Takahashi & Hideki Asada
(Hirosaki U)**

RT & Asada 2013, ApJL, 768, 16

0. Abstract

Observational constraints on cosmic abundances of negative-mass compact objects & Ellis wormholes from SDSS quasar lens survey



If there are Negative masses or Ellis wormholes in the Universe

- ➡ **Distant quasars seen as multiple images by gravitational lensing**
- ➡ **SDSS quasar survey didn't find such multiple images**
- ➡ **As a result, we can set an observational upper bound**

1. Introduction

Negative Mass Object (Bondi 1957; Jammer 1961,1999)

: Source of repulsive gravitational force

It has “negative” gravitational mass

The inertial mass can be positive or negative

$$m_I a = G \frac{m_G M_G}{r^2} \quad (\text{Eq. of Motion})$$

: Theoretical hypothetical object

: Possible ideas have been discussed since 19th century
in analogy with electric charge

: It have not been found observationally

Motion of Negative Mass in Newtonian Mechanics

- Negative mass (“negative” gravitational mass & “positive” inertial mass)
- Ordinary matter (positive gravitational & inertial masses)

▪ Negative mass and Ordinary matter



▪ Two Negative masses

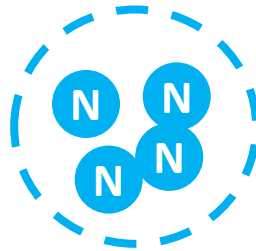
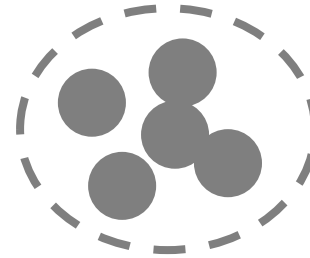
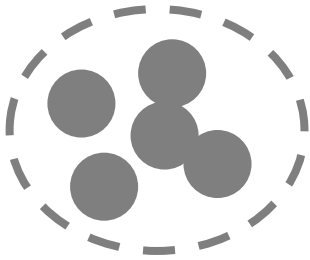


Negative-Masses Clustering in the Universe

N Negative mass

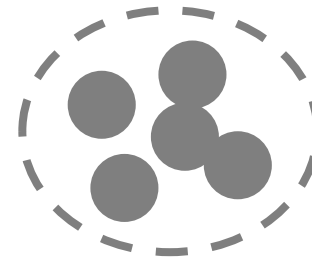
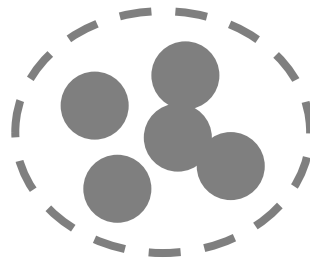
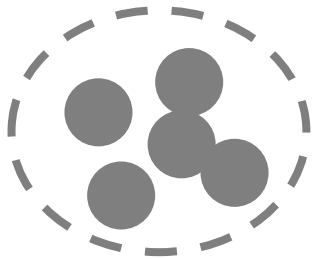
● Ordinary matter

Dark matter halo composed of ordinary matter



Negative mass clump resides in void

(e.g. Piran 1997)



Motion of Negative Mass in Newtonian Mechanics

- Negative mass (“negative” gravitational mass & “negative” inertial mass)
- Ordinary matter (positive gravitational & inertial masses)

▪ Ordinary matter and Negative mass



▪ Two Negative masses

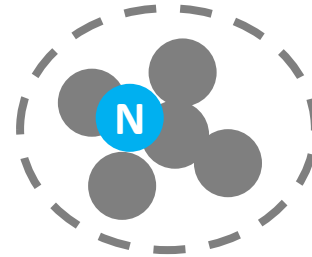
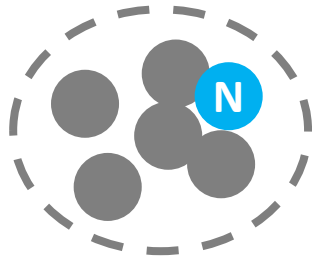


Negative-Masses Clustering in the Univesers

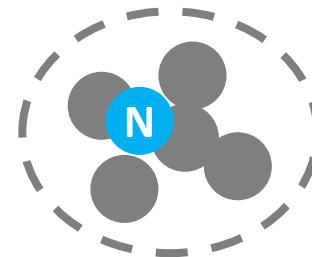
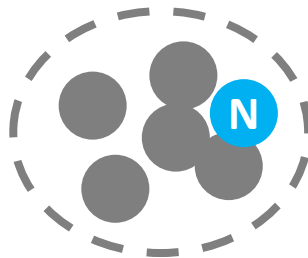
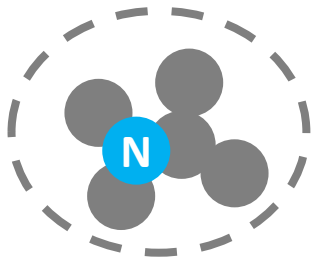
 Negative mass

 Ordinary matter

Dark matter halo composed
of ordinary matter

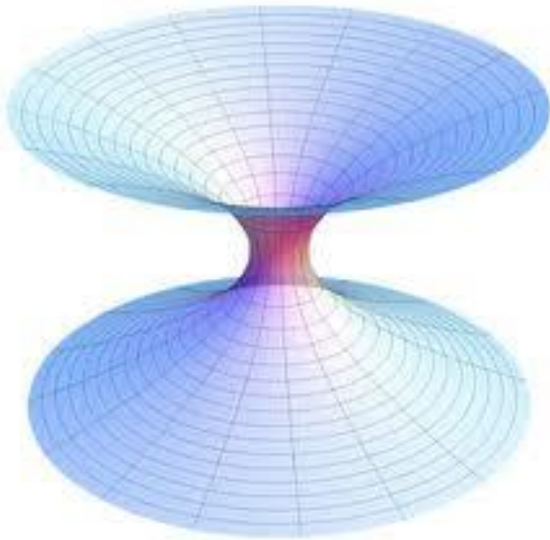


Negative Masses exist in Dark Haloes



Wormhole

(Morris & Thorne 1988; Morris+ 1988)



**“Tunnel” connects distant space-time
theoretical prediction of general relativity
A solution of Einstein Eq.**

Ellis wormhole

(Ellis 1973)

- a solution of Einstein eq.
- massless scalar field

Metric

a : throat radius

$$ds^2 = dt^2 - dr^2 - (r^2 + a^2)(d\theta^2 + \sin^2\theta d\varphi^2)$$

no interaction with matters

no light emission

light ray path is deflected by gravitational lensing

**By studying the negative masses or Ellis wormholes,
we might obtain answers of fundamental questions in Physics :**

Q. Why is gravitational force always attractive?

**Q. Why is particle (inertial & gravitational) mass
always positive?**

Previous upper bound on abundance of negative masses

Cramer+ 1995

Gravitational microlensing by
negative masses in our galaxy

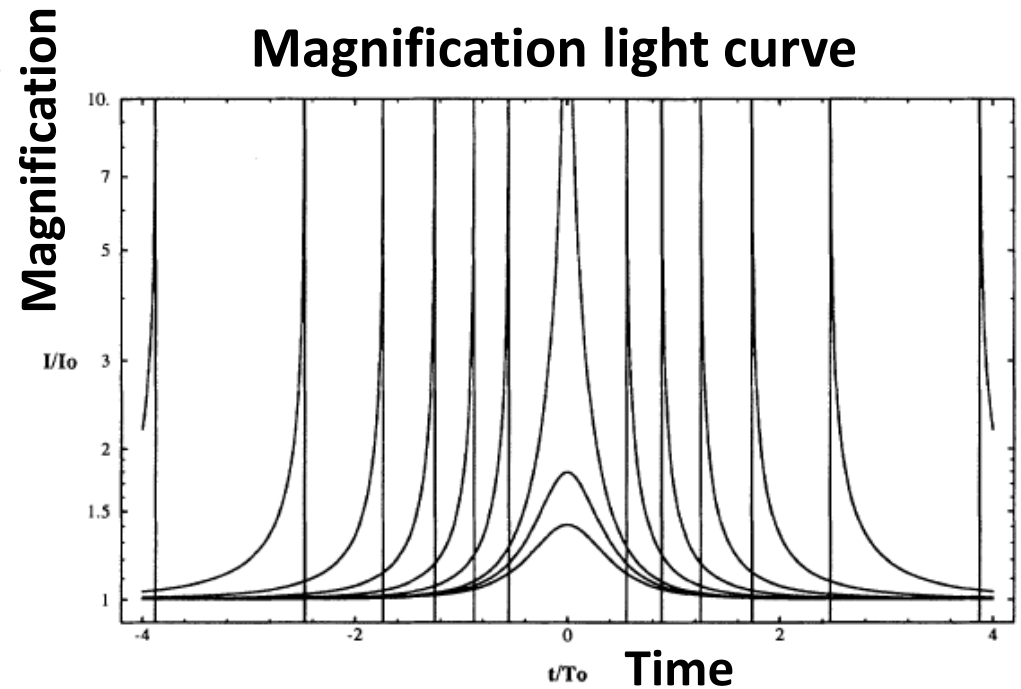


FIG. 3. Intensity profile of a gravitationally negative anomalous compact halo object (GNACHO) as it passes near the source-detector axis DS . The several curves correspond to minimum dimensionless impact parameter values $B_0 = 0.50$ (at edge of plot), 0.75, 1.00, 1.25, 1.50, 1.75, 2.00, 2.10, and 2.20 (small central bump). (See text for definitions of the variables.)

Previous upper bound on abundance of negative masses

Torres+ 1998

Gravitational lensing of distant AGNs(Active Galactic Nuclei) by negative masses may be detected

- ➡ Such a lensing event has not been detected
- ➡ Set an upper bound on the abundance of negative masses

$$|\rho| < 10^{-36} \text{ g cm}^{-3} \quad \text{for } |M| \approx 0.1M_{\odot}$$

$$|\Omega| < 10^{-7} \quad (\text{cosmological density parameter})$$

Previous upper bound on abundance of negative masses

Safonova+ 2001

Microlensing of finite-size source

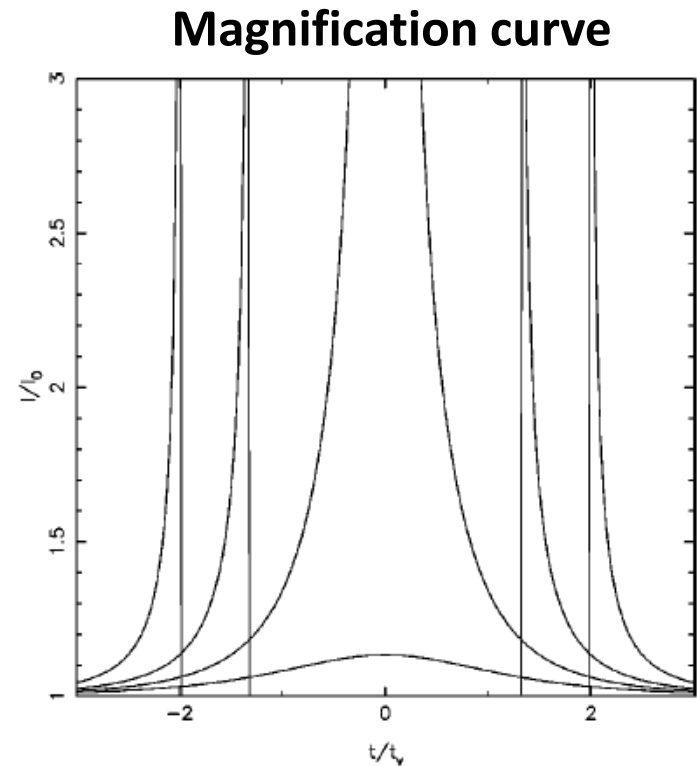
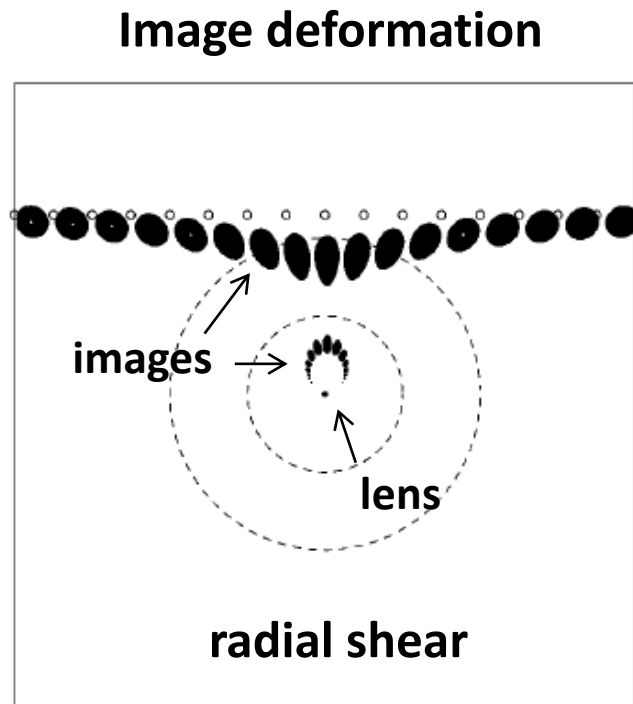


FIG. 6. True motion of the source and apparent motion of the images for $B_0 > 2$. The inner dashed circle is the Einstein ring, the outer dashed circle is twice the Einstein ring. The rest is as in Fig. 5.

FIG. 10. Light curves for the negative mass lensing of a point source. From the center of the graph towards the corners the curves correspond to $B_0 = 2.5, 2.0, 1.5, 0.0$. The time scale here is ξ_0 divided by the effective transverse velocity of the source.

Previous upper bound on abundance of Ellis wormhole

Abe 2010

Microlensing in
our galaxy

Constraint on wormhole
with throat radius

$$a = 100 - 10^7 \text{ km}$$

Magnification light curve

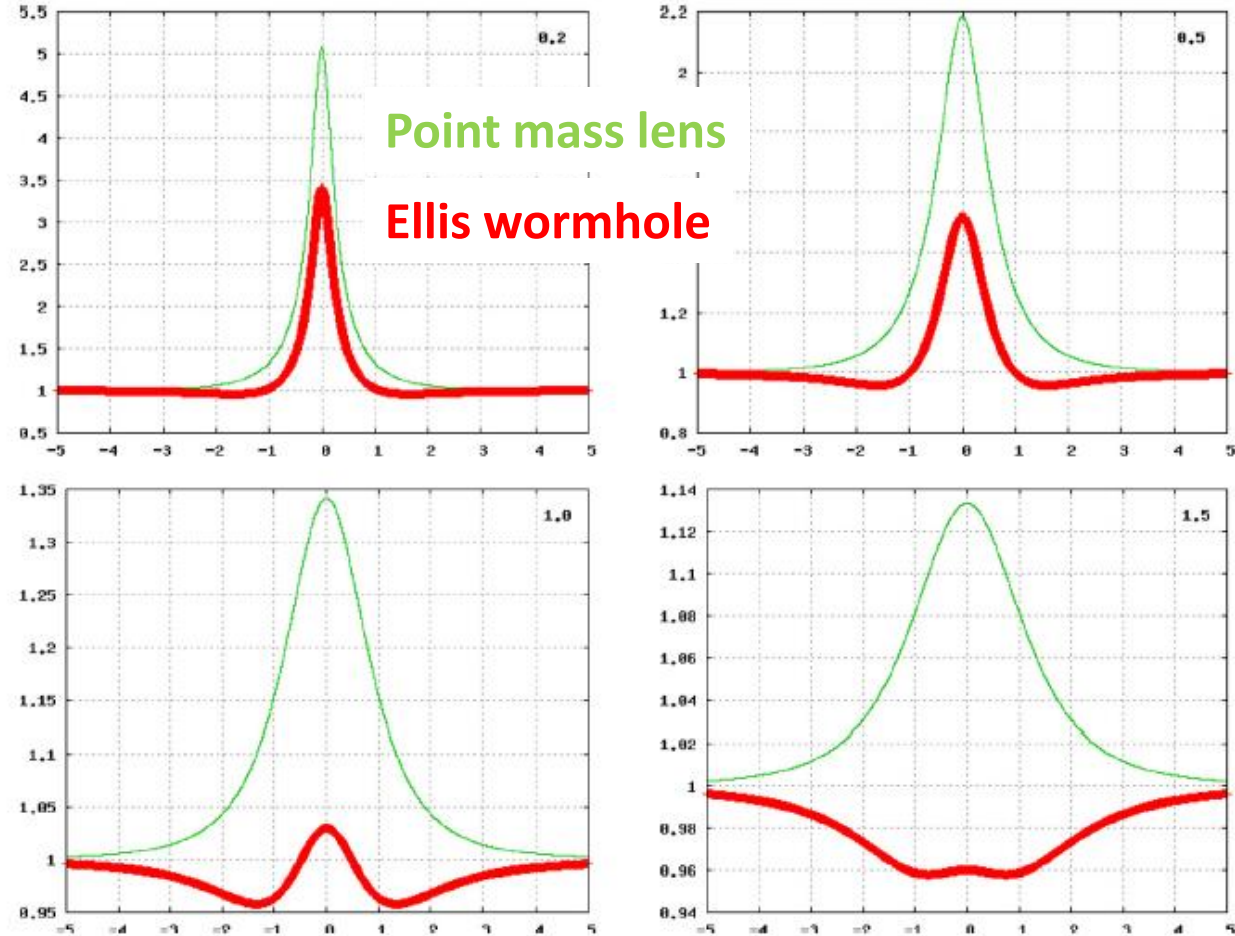


Figure 4. Light curves for $\hat{\beta}_0 = 0.2$ (top left), $\hat{\beta}_0 = 0.5$ (top right), $\hat{\beta}_0 = 1.0$ (bottom left), and $\hat{\beta}_0 = 1.5$ (bottom right). Thick red lines are the light curves for wormholes. Thin green lines are corresponding light curves for Schwarzschild lenses.
(A color version of this figure is available in the online journal.)

Previous upper bound on abundance of Ellis wormhole

Yoo+ 2013

Gravitational lensing of distant GRBs(Gamma Ray Bursts) by wormholes

- ➔ Such an event has not been detected (**Barnacka+ 2012**)
- ➔ Give an upper bound on its number density

$$n < 10^{-9} \text{AU}^{-3} \quad \text{for } a \approx 0.1 \text{cm}$$

Observational data we used

SDSS (Sloan Digital Sky Survey) Quasar Lens Search

Largest homogeneous sample of Quasars

(Oguri+ 2006,2008,2012; Inada+ 2012)

- **# of quasars 50836**
- **redshifts $z=0.6-2.2$**
- **apparent magnitude <19.1 (i band)**
- **Lensed quasar systems 19 (image separations 1-20arcsec)**
- **No multiple image lensed by negative masses and Ellis wormholes**

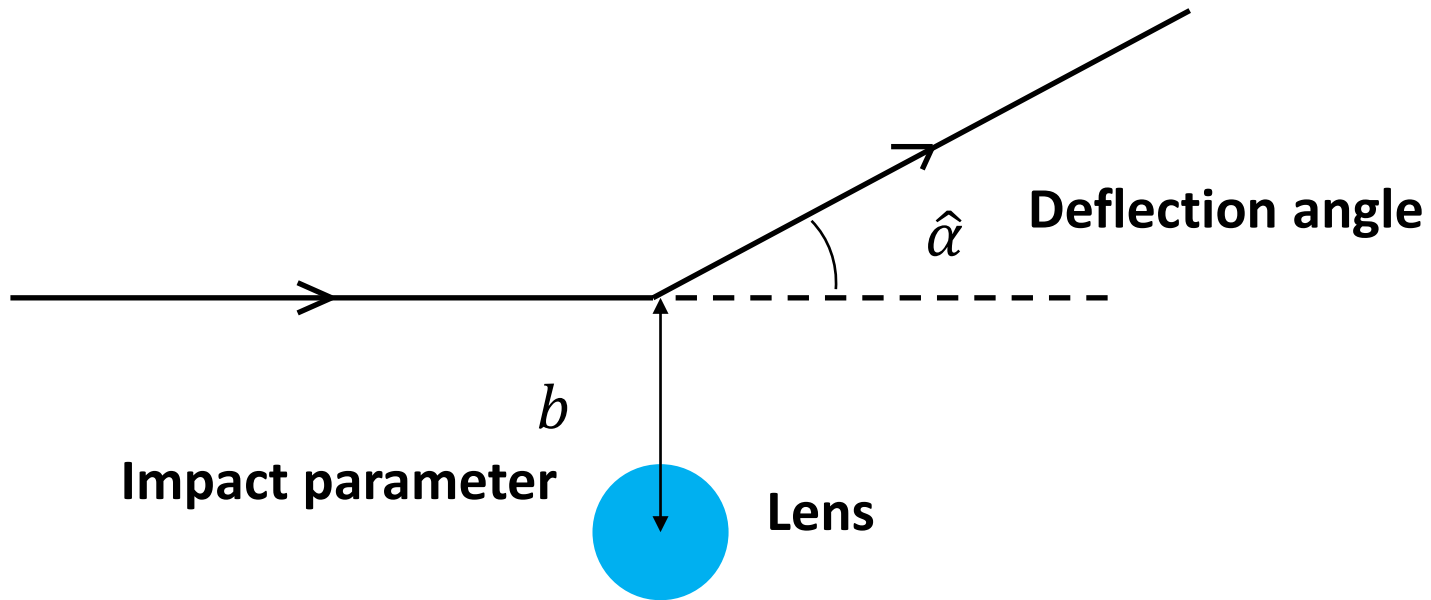
(Oguri & Kayo 2012, private communication)

Lensing by Negative Mass

(Cramer+ 1995; Safonova+ 2001)

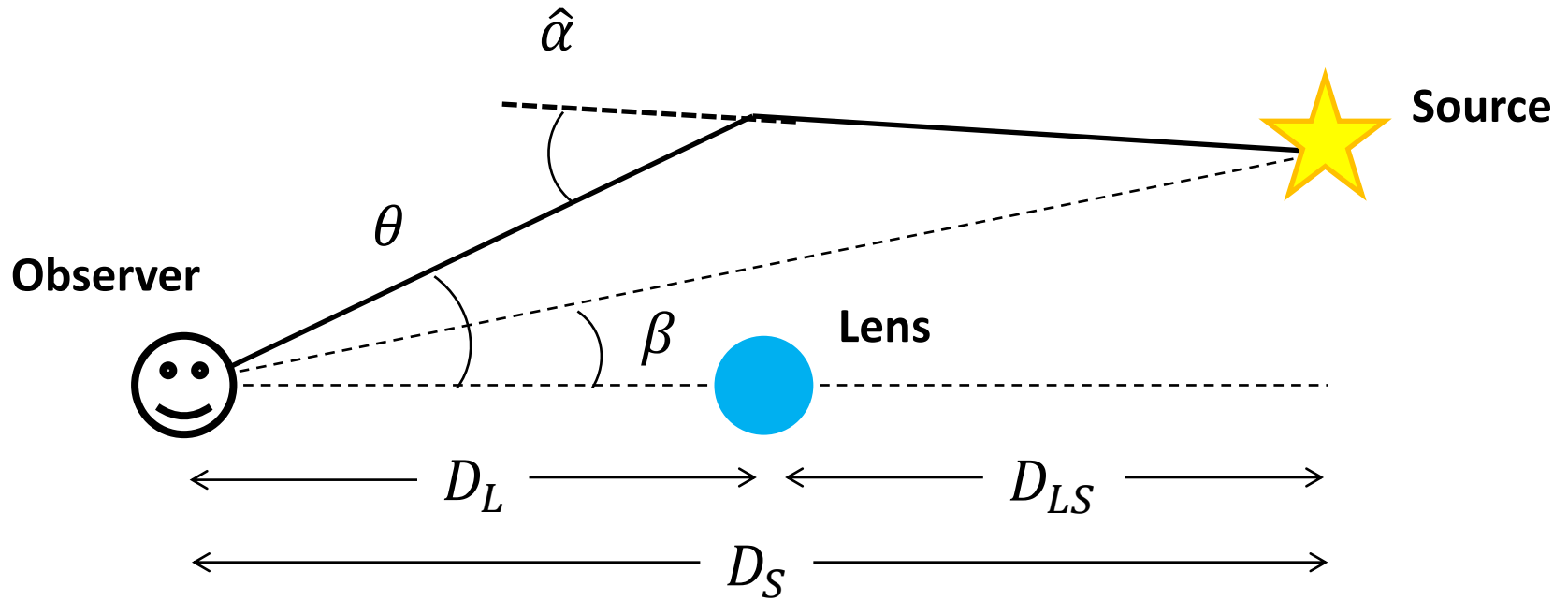
Negative point mass ($M < 0$)

Deflection angle is same as ordinary point mass lens,
but its sign is opposite



$$\hat{\alpha} = \frac{4|M|}{b}$$

Lens Equation

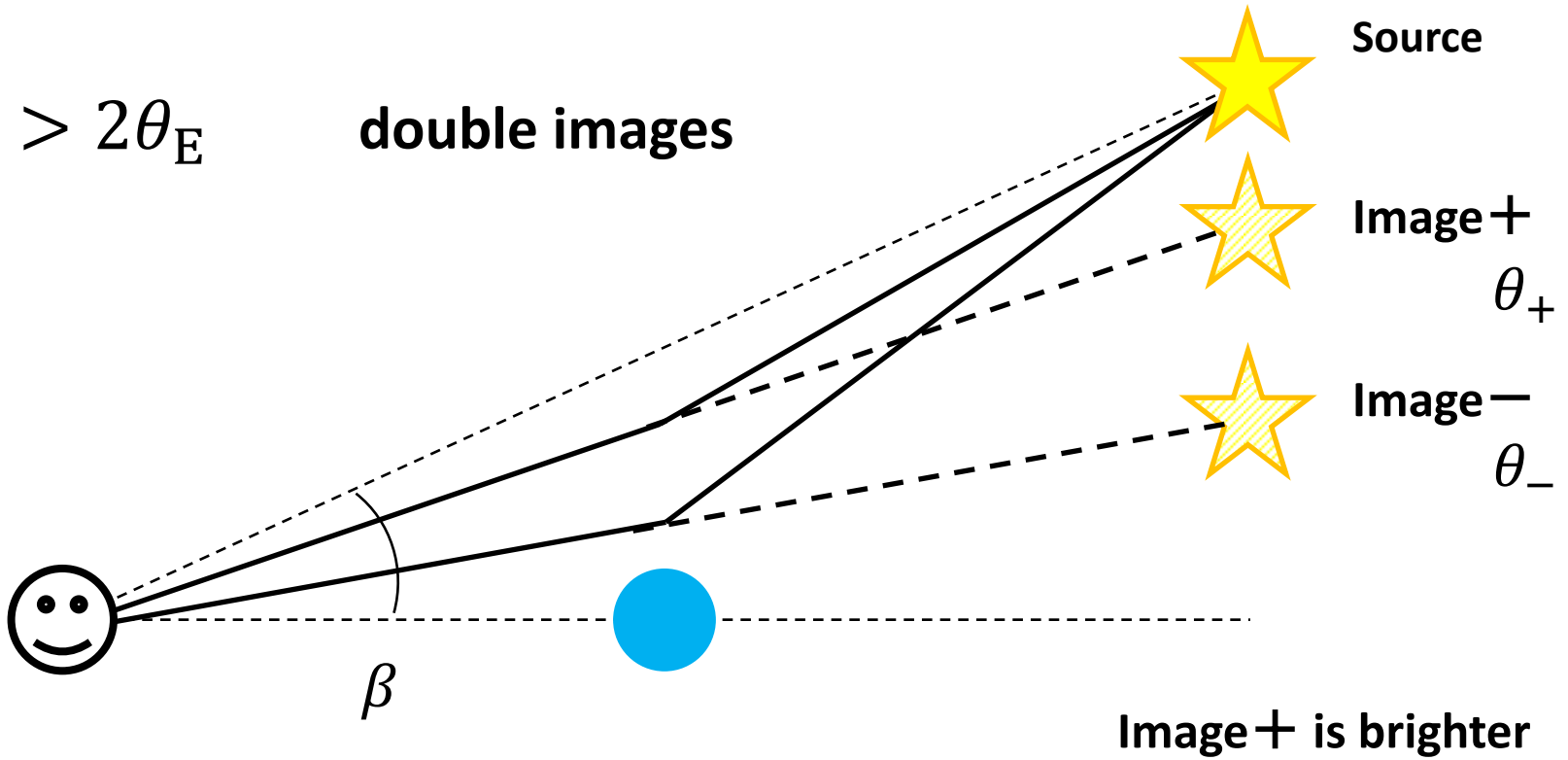


$$\beta = \theta - \frac{D_{LS}}{D_S} \hat{\alpha}$$

of images

$$\beta > 2\theta_E$$

double images

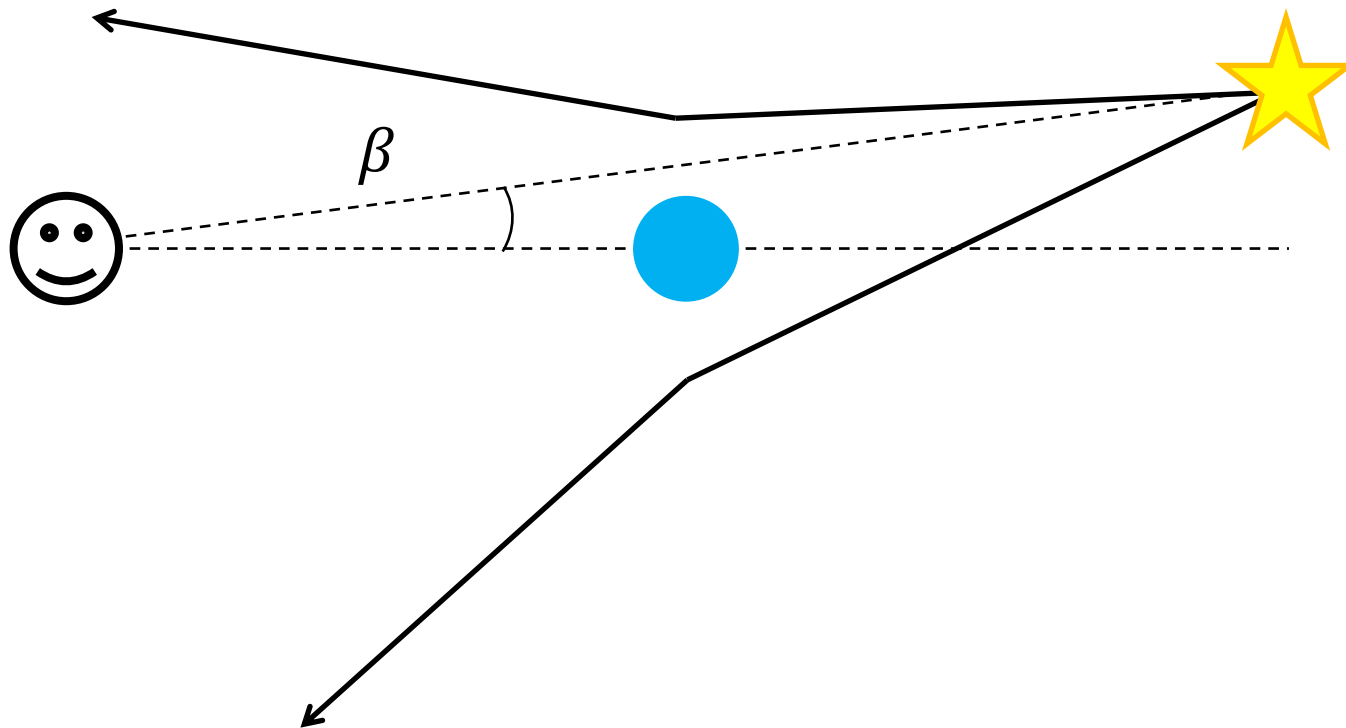


$$\theta_{\pm} = \frac{1}{2} \left(\beta \pm \sqrt{\beta^2 - 4\theta_E^2} \right)$$

of images

$$\beta < 2\theta_E$$

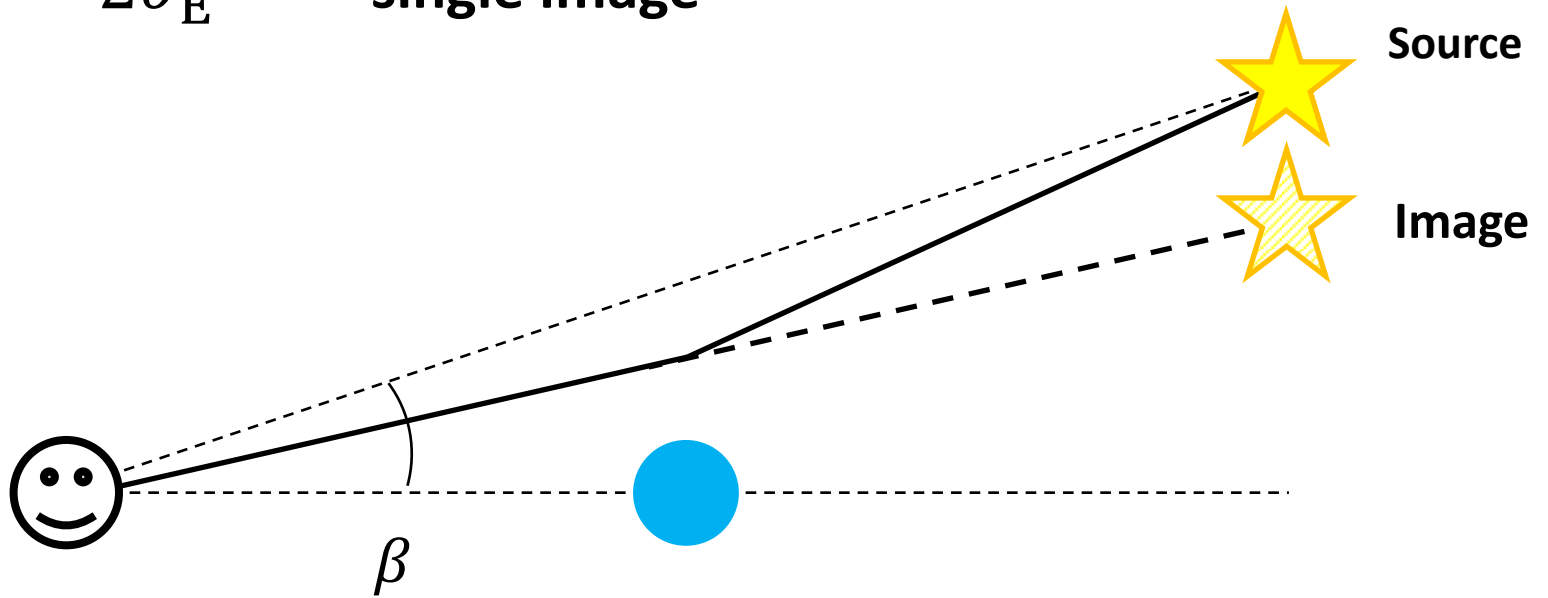
zero image



of images

$$\beta = 2\theta_E$$

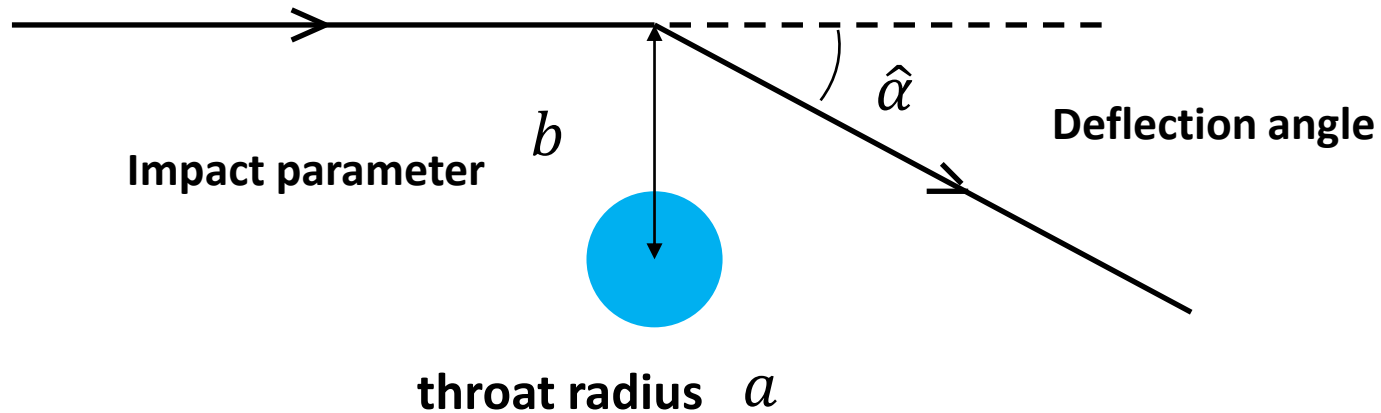
single image



Infinite magnification

Gravitational lensing by Ellis wormhole

(Clement 1984; Dey & Sen 2008; Toki+ 2011; Nakajima & Asada 2012)



$$\hat{\alpha} = \frac{\pi a^2}{4 b^2}$$

Lens equation

$$\beta = \theta - \theta_E^3 \frac{\theta}{|\theta|^3}$$

Einstein radius

$$\theta_E = \left(\frac{\pi a^2}{4} \frac{D_{LS}}{D_L^2 D_S} \right)^{1/3}$$

 Double images form

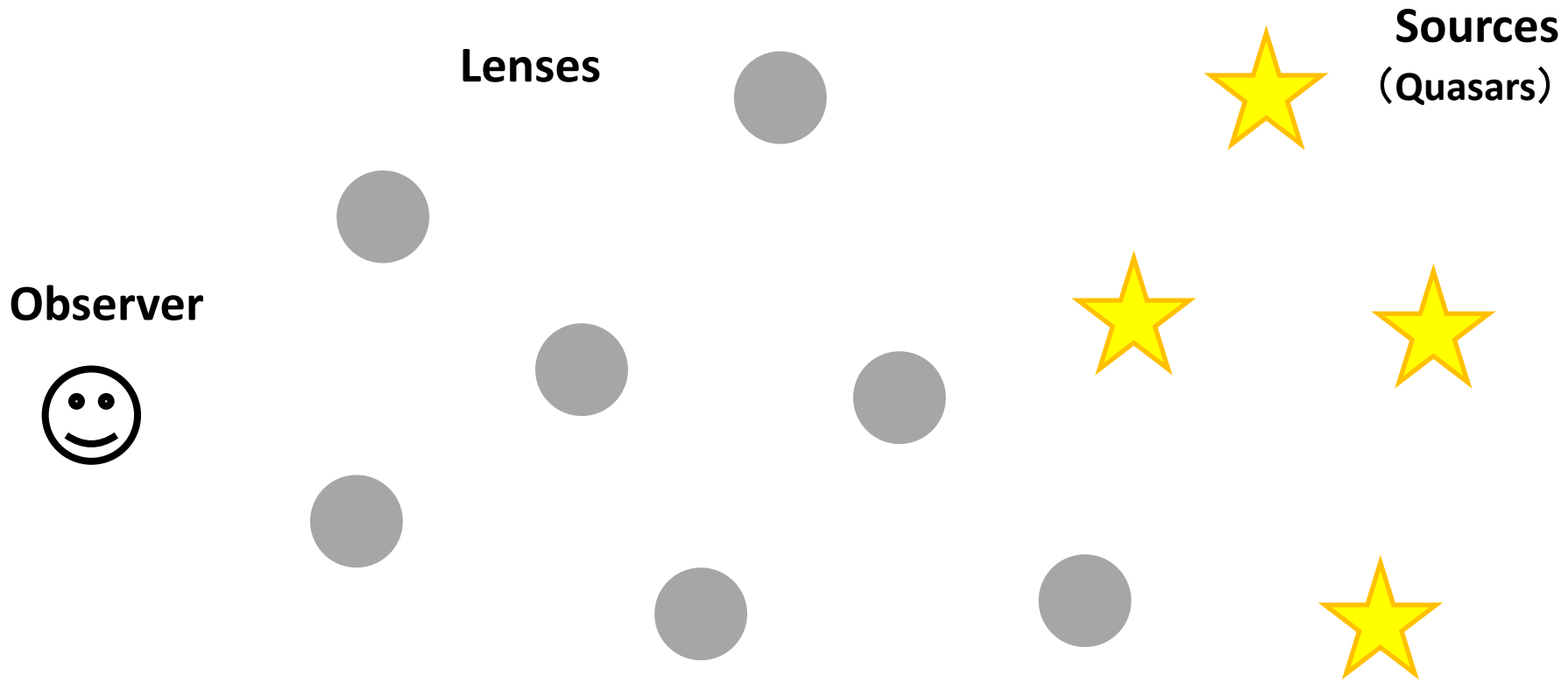
throat radius estimated from Einstein radius

$$a = 10 h^{-1} \text{pc} \left(\frac{\theta_E}{1''} \right)^{3/2} \left[\frac{D_L D_S / D_{LS}}{(1 h^{-1} \text{Gpc})^2} \right]^{1/2}$$

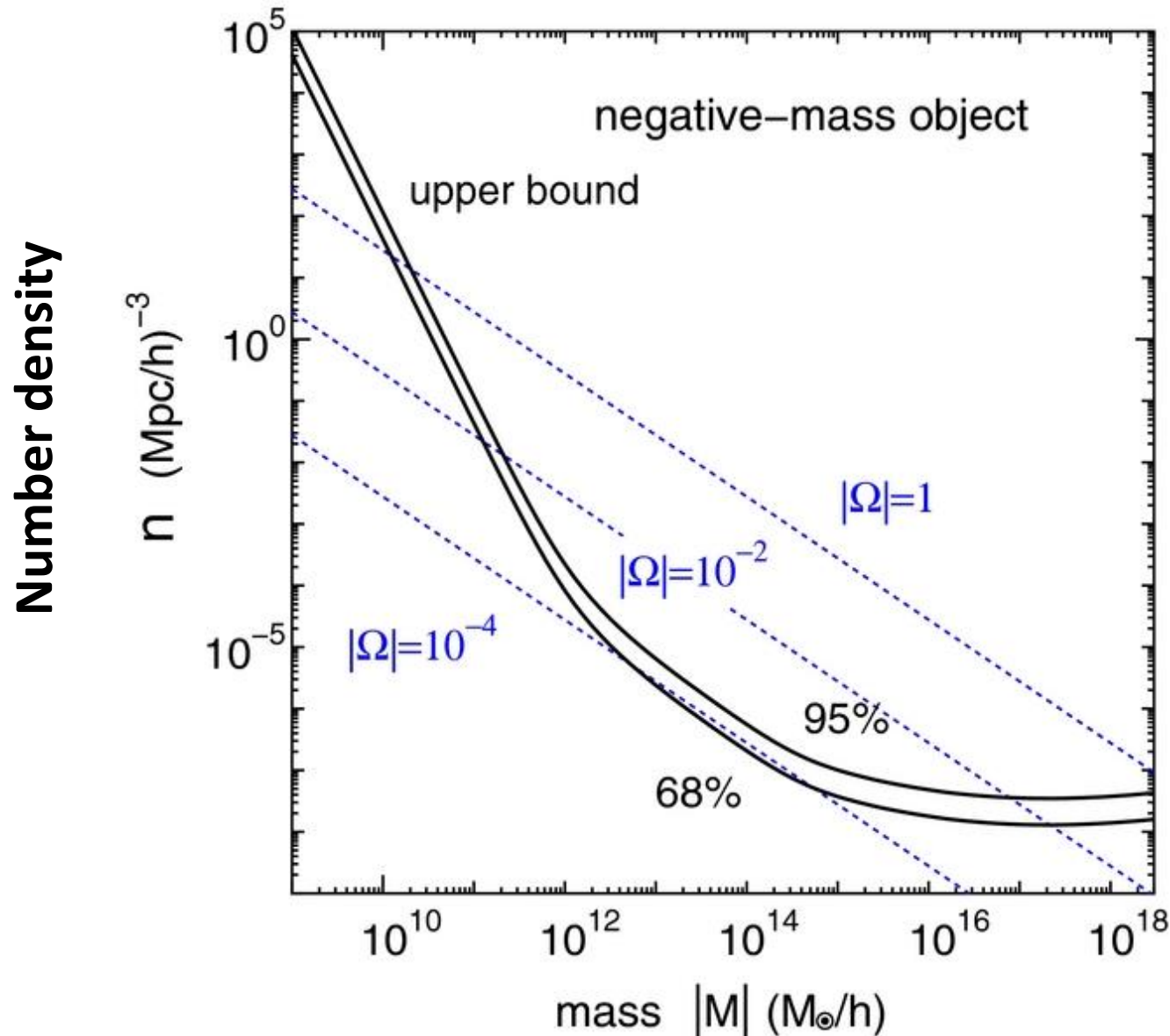
throat radius $a = 10 - 100 \text{pc}$

Negative Masses & Ellis Wormholes are distributed homogeneous

Number density of lenses n \rightarrow Lensing probability $\propto n$



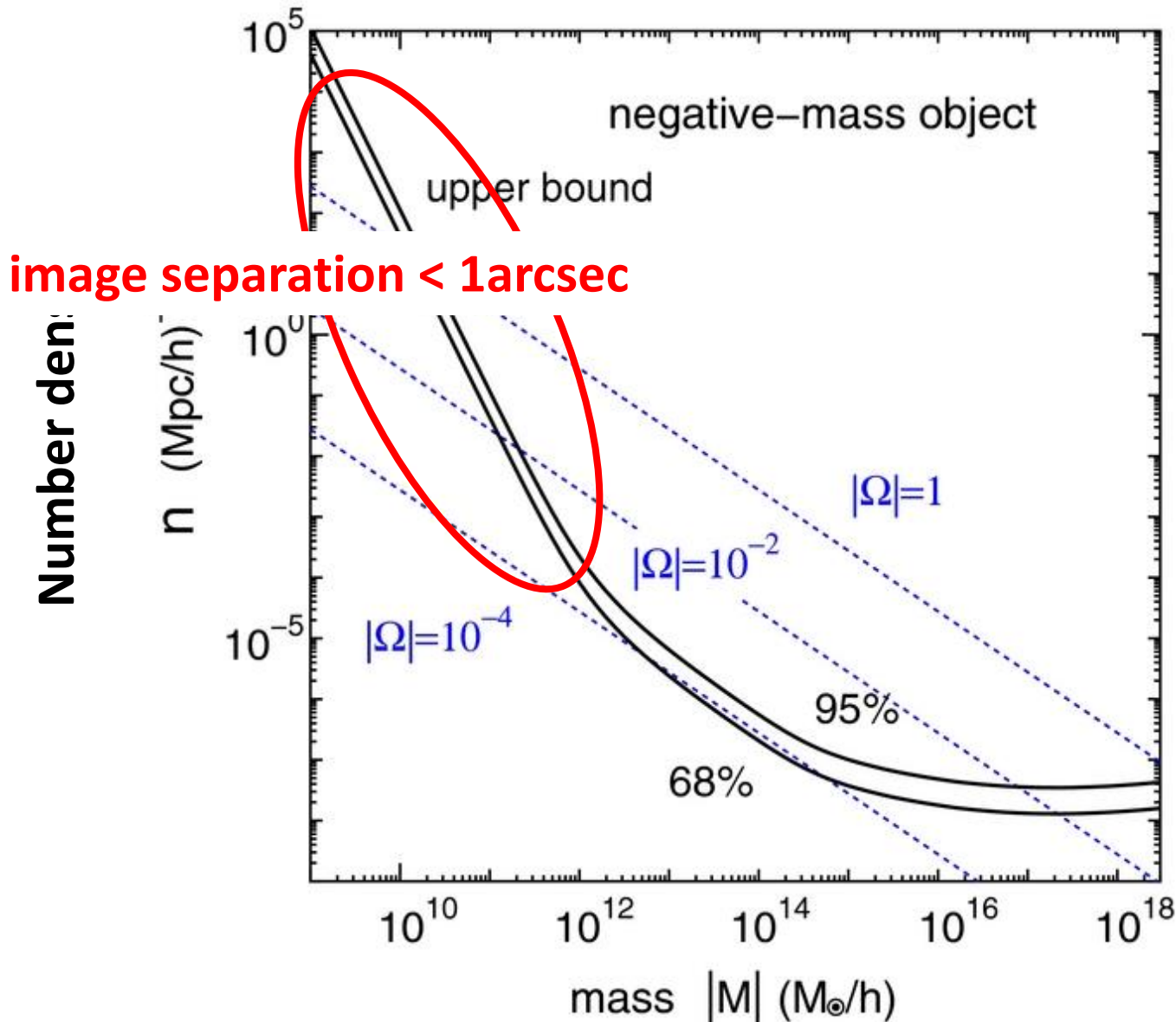
Upper Bound on Cosmological Number Density of Negative Mass



$$\Omega = \frac{Mn}{\rho_{\text{crit}}}$$

(cosmological density parameter)

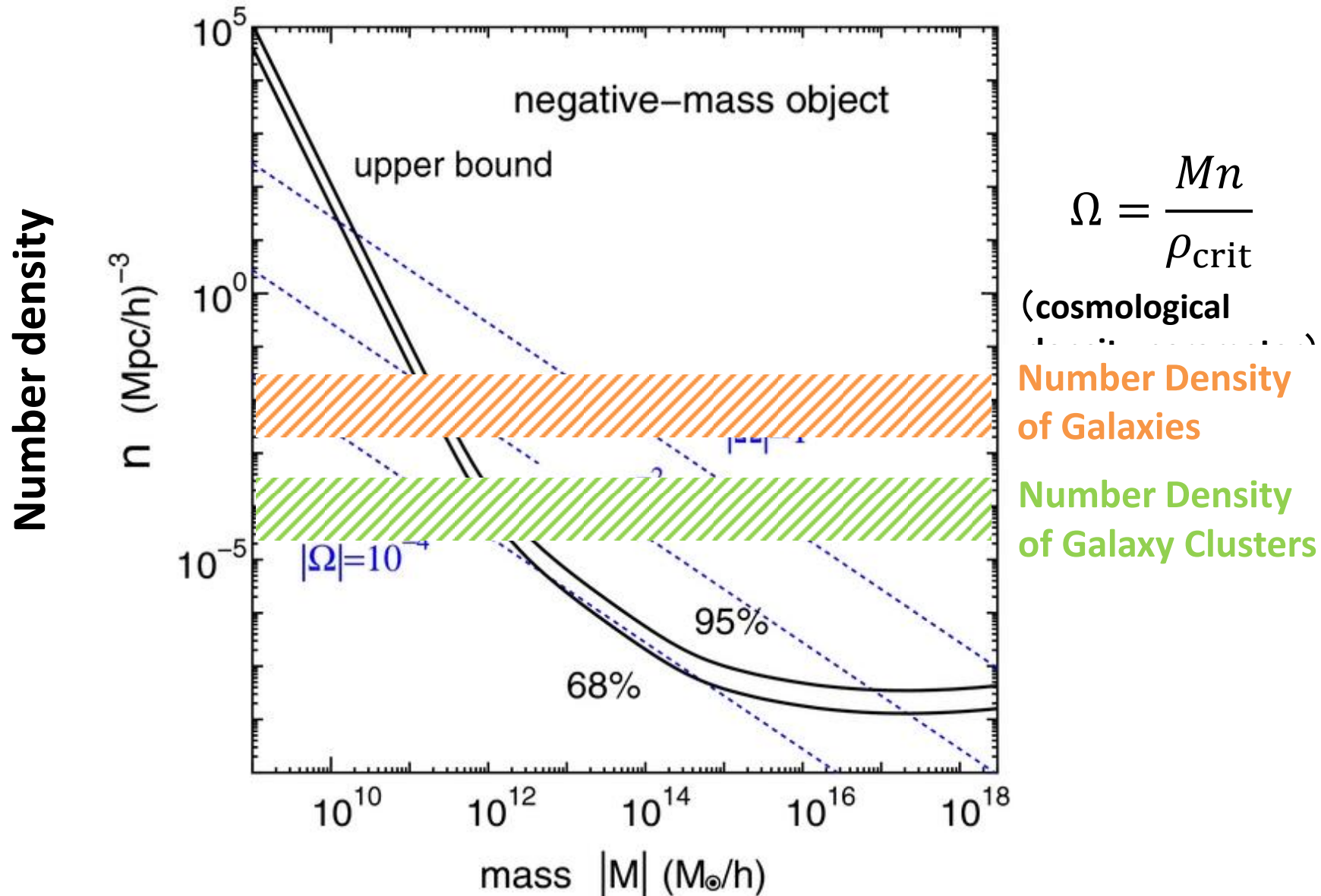
Upper Bound on Cosmological Number Density of Negative Mass



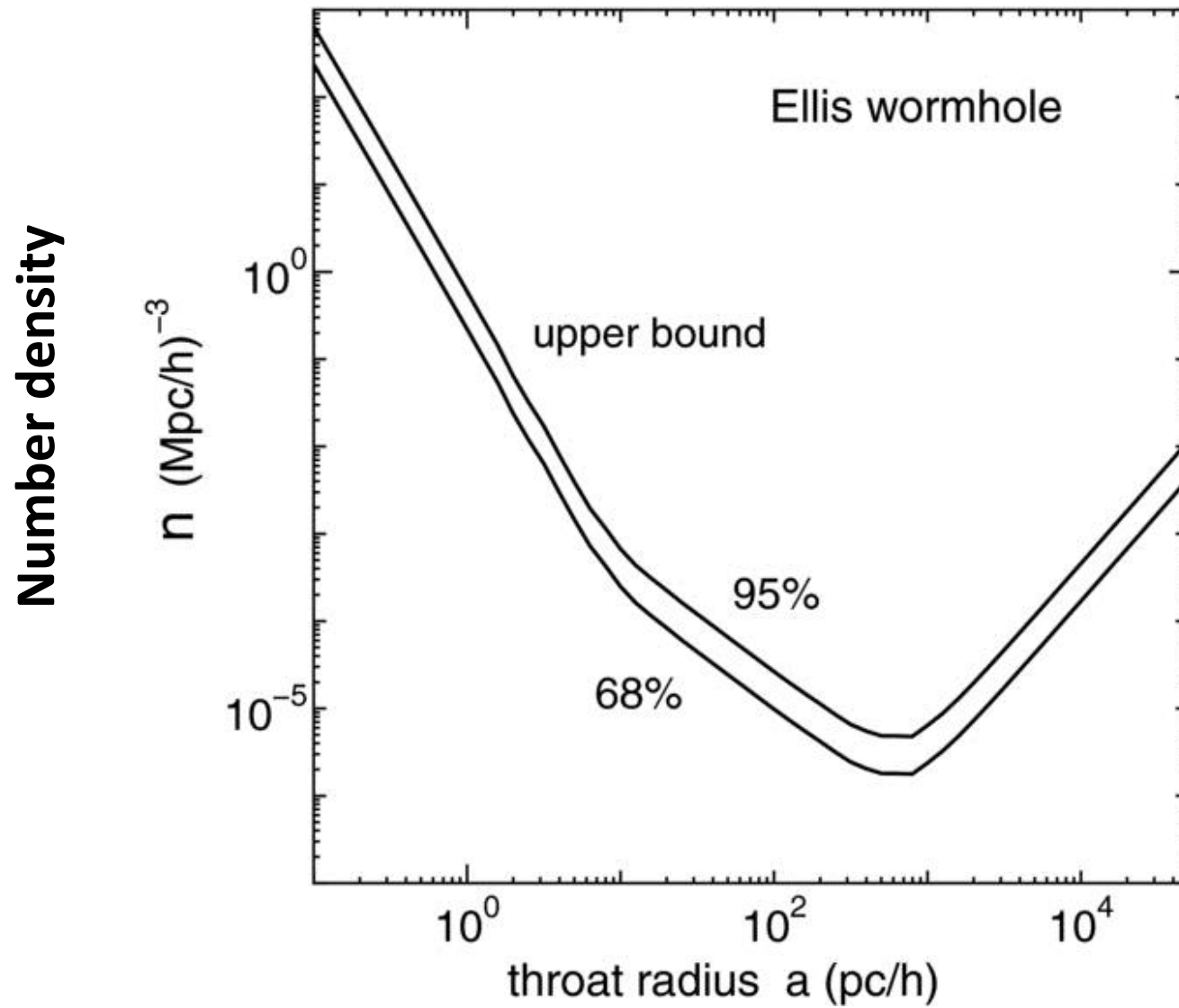
$$\Omega = \frac{Mn}{\rho_{\text{crit}}}$$

(cosmological
density parameter)

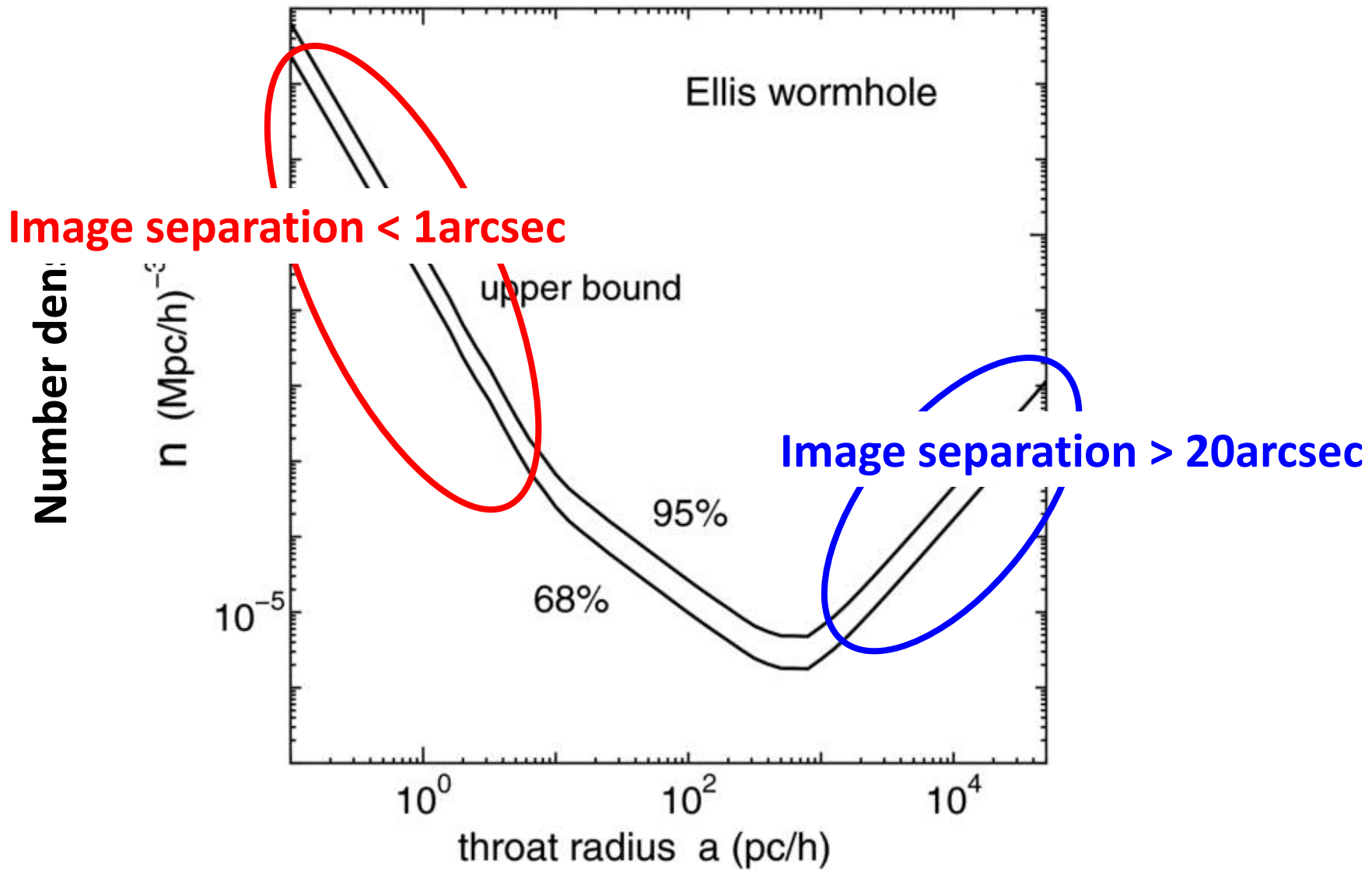
Upper Bound on Cosmological Number Density of Negative Mass



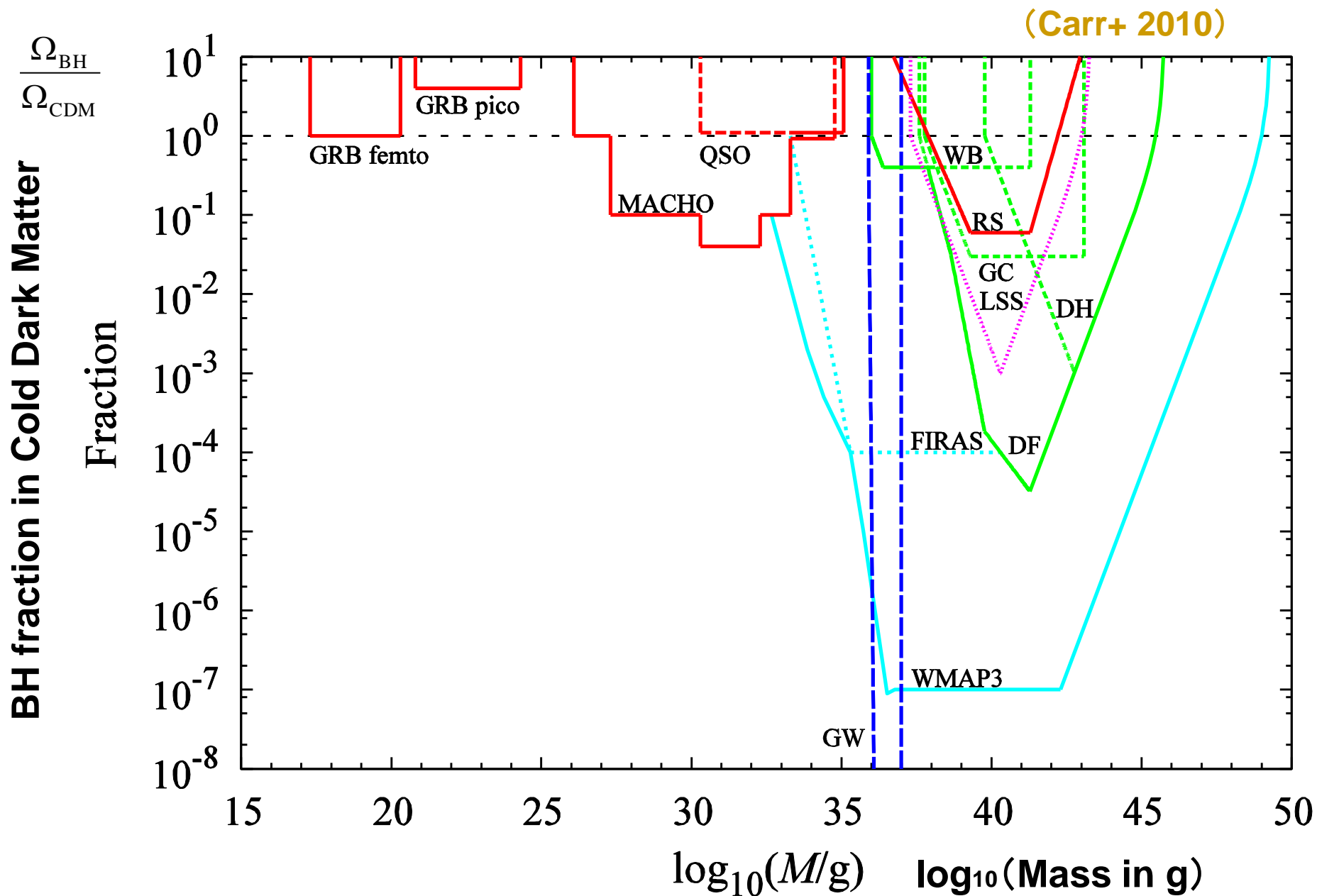
Upper Bound on Cosmological Number Density of Ellis Wormhole



Upper Bound on Cosmological Number Density of Ellis Wormhole



Observational Upper Bound on Black Hole Abundance



Search Negative Masses in Cosmic Voids via Weak Lensing

Negative masses may reside in cosmic voids

Density contrast

$$\delta = \frac{\rho}{\bar{\rho}} - 1$$

If $\delta < -1$ evidence of negative masses

Under dense region (convergence <0 or $\delta <0$) already measured

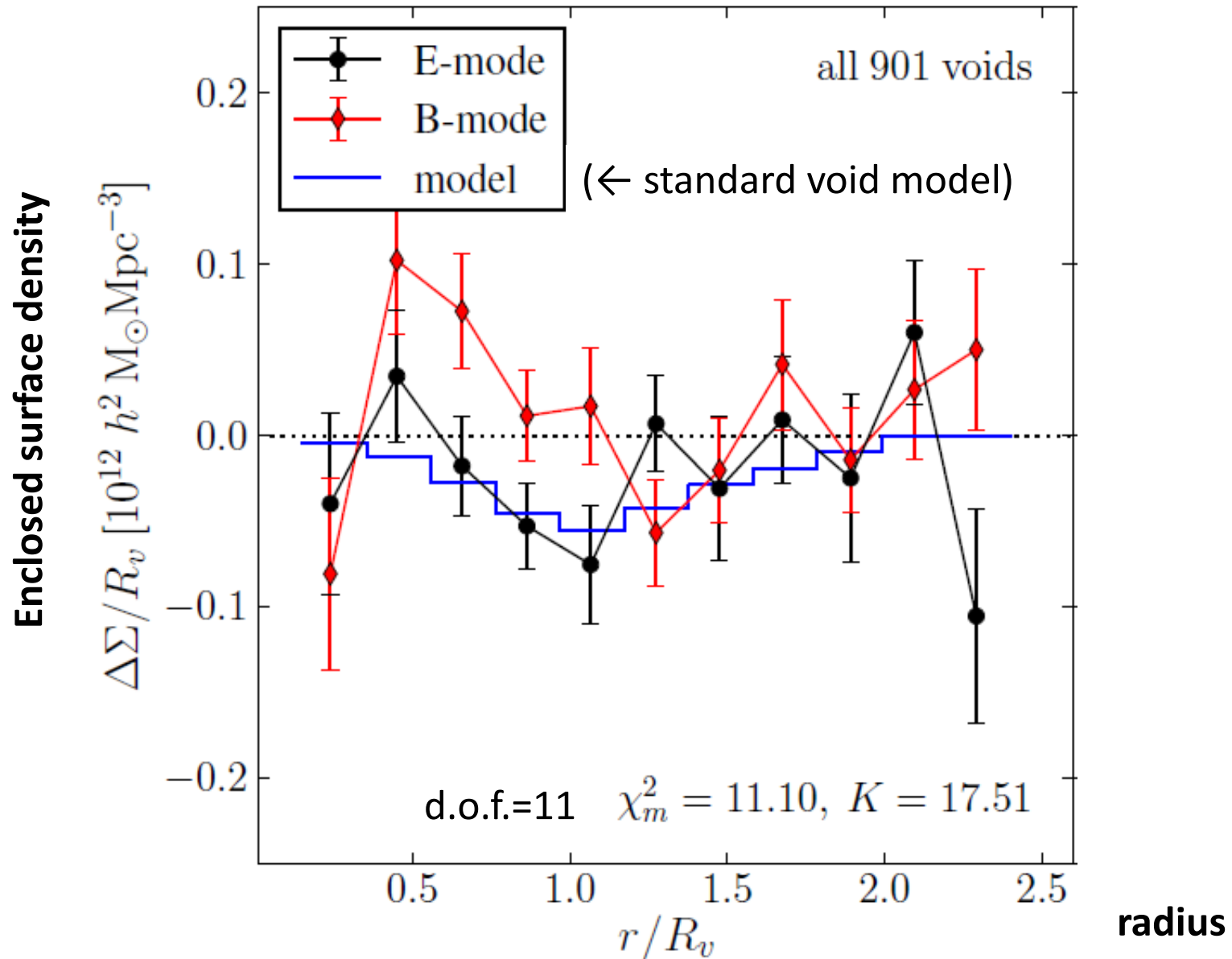
(Miyazaki+ 2002; Gavazzi & Soucail 2007; Shan+ 2012)

Future surveys (Subaru HSC etc.) will measure its density profile

(Higuchi+ 2013; Krause+ 2013)

First measurement of void density profiles in SDSS

(Melchior+ 2013, arXiv:1309.2045)



Summary

- **Negative Mass Object**

$$n < 10^{-8} (10^{-4}) h^3 \text{Mpc}^{-3}$$

for mass $|M| > 10^{15} (10^{12}) M_{\odot}$

$$|\Omega| < 10^{-4} \quad \text{for mass } M = 10^{12-14} M_{\odot}$$

- **Ellis Wormhole**

$$n < 10^{-4} h^3 \text{Mpc}^{-3}$$

for throat radius $a = 10 - 10^4 \text{pc}$

2重像のなす角 ~ Einstein radius

$$\theta_E = \sqrt{4|M| \frac{D_{LS}}{D_L D_S}}$$

レンズ質量の推定

$$|M| = 1 \times 10^{11} h^{-1} M_{\odot} \left(\frac{\theta_E}{1''} \right)^2 \left[\frac{D_L D_S / D_{LS}}{1 h^{-1} \text{Gpc}} \right]$$

SDSSで探す像のなす角 1-20 arcsec

10^{11-14} 太陽質量の負質量天体を探査

例：等温球型 SIS (Singular Isothermal Sphere)

密度分布

$$\rho(r) = \frac{\sigma^2}{2\pi} \frac{1}{r^2}$$

σ : 速度分散

レンズ方程式

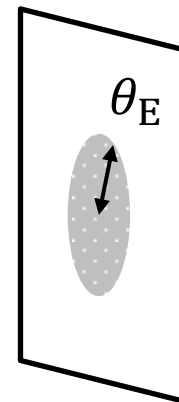
$$\beta = \theta \pm \theta_E$$

Einstein radius

$$\theta_E = 4\pi\sigma^2 \frac{D_{LS}}{D_S}$$

➡ $\beta < \theta_E$ なら2重像

$$\text{確率} = \frac{\pi\theta_E^2}{4\pi}$$



SDSS観測データとの比較

赤方偏移 z 、等級 m の光源が多重像を作る確率 $P(z, m)$

SDSSでの全光源数 $N_Q=50,836$

負質量天体やワームホールにより重力レンズを受けた光源は一つも受かっていない

(多重像は見えているが、レンズ天体が見えていないもの)

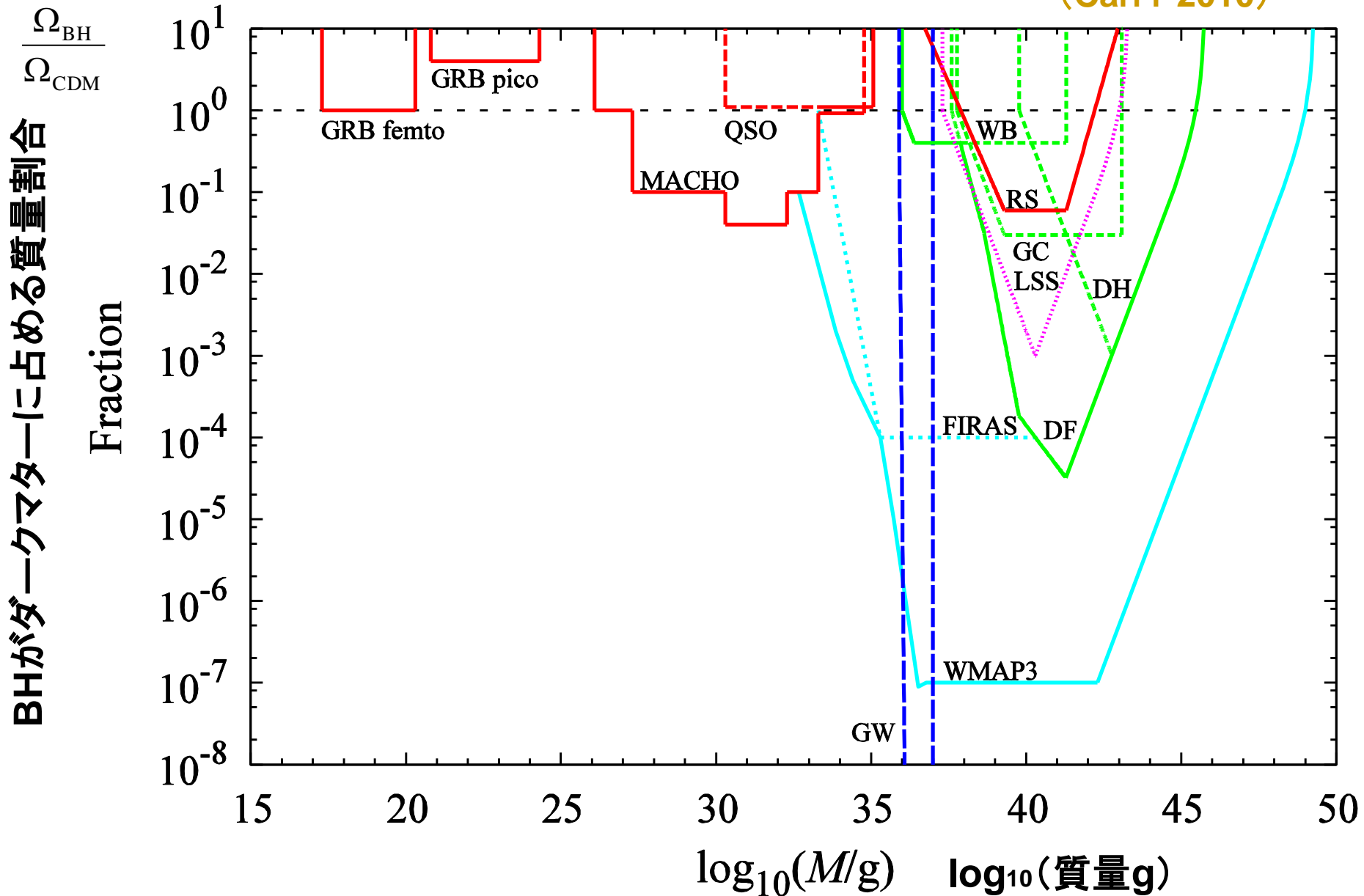
受からない確率

$$P_{\text{tot}} = \prod_{i=1}^{N_Q} (1 - P(z_i, m_i))$$

BH (black hole) の存在量

観測からの上限

(Carr+ 2010)



これまでの負質量天体の観測的制限

Safonova+ 2001

有限な大きさを持った光源のマイクロレンズ

像の歪み

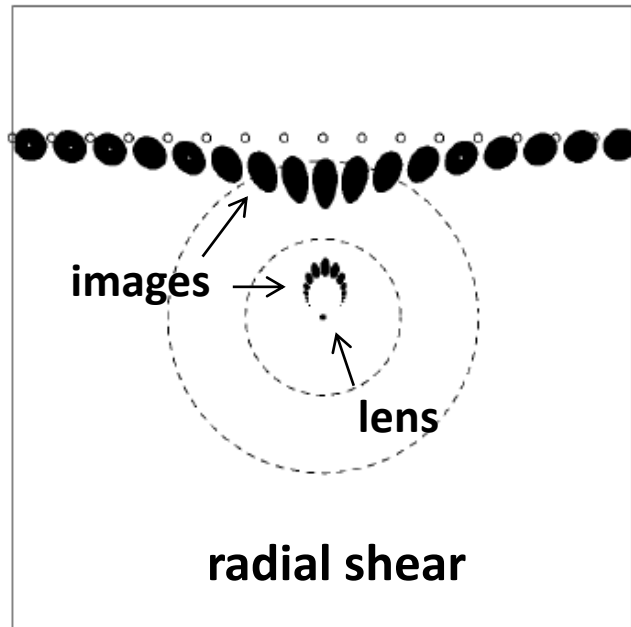


FIG. 6. True motion of the source and apparent motion of the images for $B_0 > 2$. The inner dashed circle is the Einstein ring, the outer dashed circle is twice the Einstein ring. The rest is as in Fig. 5.

増光曲線

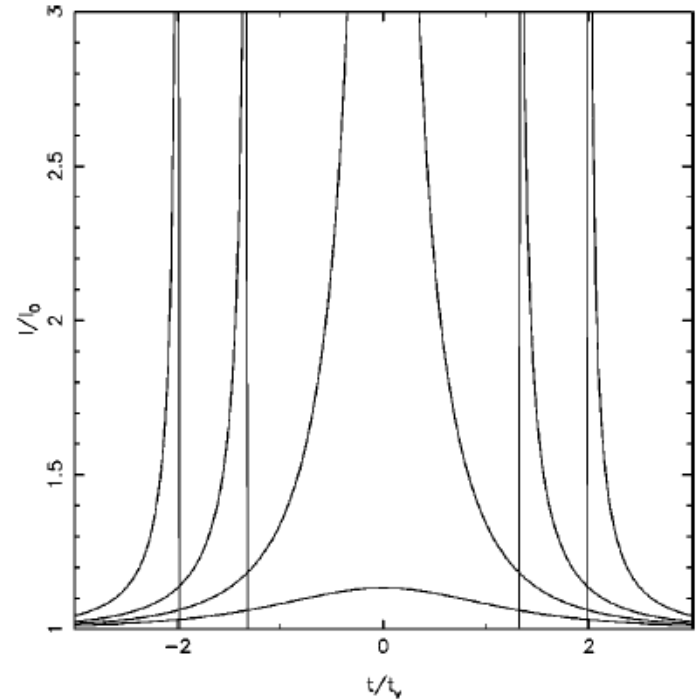
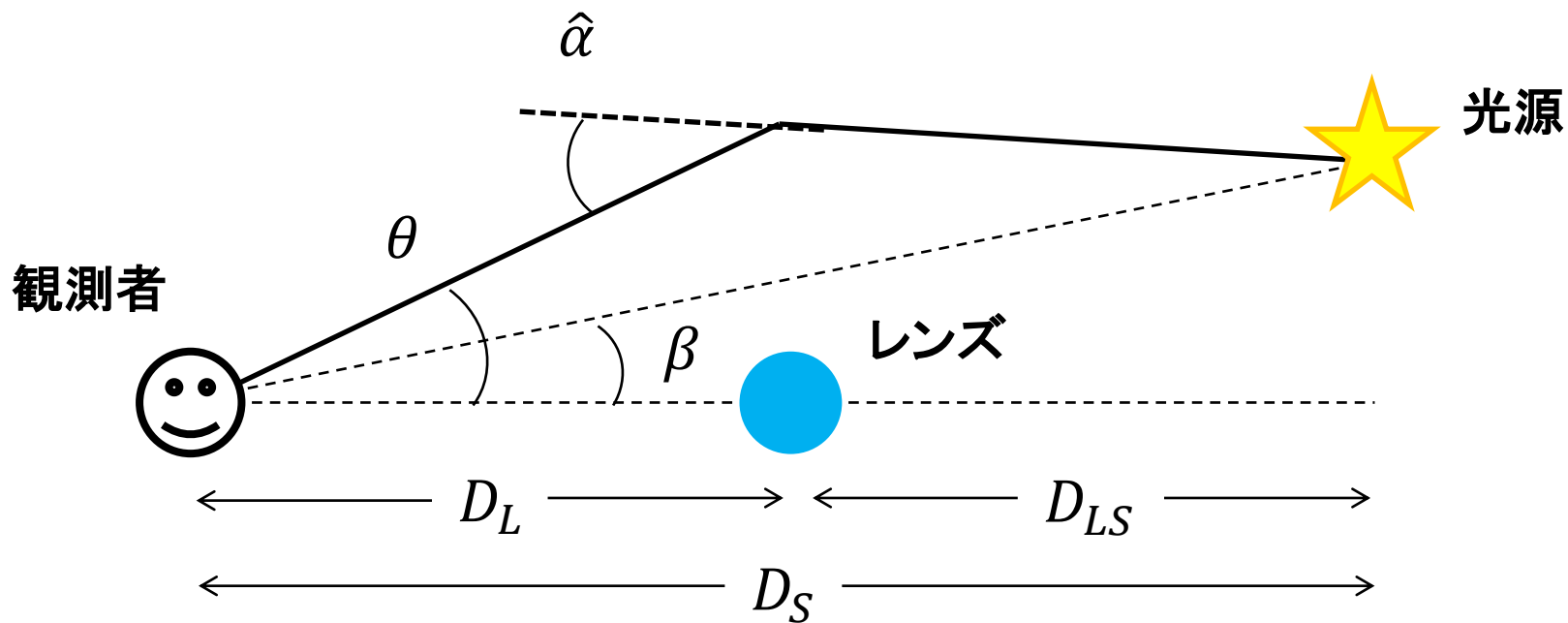


FIG. 10. Light curves for the negative mass lensing of a point source. From the center of the graph towards the corners the curves correspond to $B_0 = 2.5, 2.0, 1.5, 0.0$. The time scale here is ξ_0 divided by the effective transverse velocity of the source.

レンズ方程式



$$\beta = \theta - \frac{D_{LS}}{D_S} \hat{\alpha}$$

SDSSで多重像として観測されるための条件

- ・ 2重像のなす角は 1-20 arcsec
- ・ 2重像の明るさの比は $10^{0.5}$ (=3.2) 以下
- ・ 明るさは 19.1 等級 (i band) 以下

負質量天体の振舞い

～ 慣性質量 <0 & 重力質量 <0 の場合 ～

- 普通の物質 (慣性質量 >0 & 重力質量 >0)
- 負質量物質 (慣性質量 <0 & 重力質量 <0)

負質量物質 同士

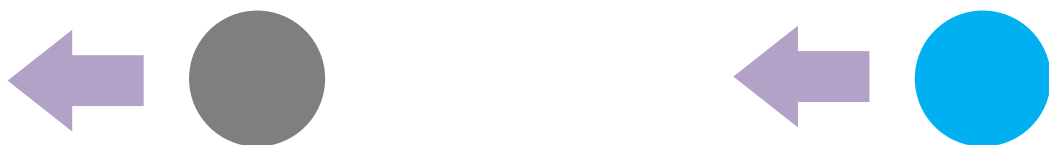


互いに反発しあう

負質量天体の振舞い

～ 慣性質量 <0 & 重力質量 <0 の場合 ～

- 普通の物質（慣性質量 >0 & 重力質量 >0 ）
- 負質量物質（慣性質量 <0 & 重力質量 <0 ）



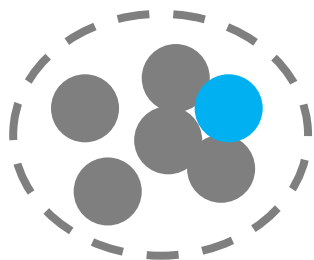
負質量物質が普通の物質を追いかける

負質量天体の振舞い

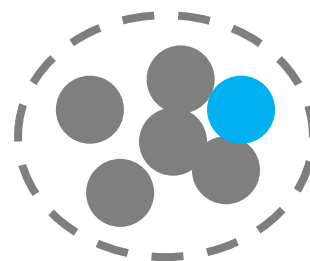
～ 慣性質量 <0 & 重力質量 <0 の場合 ～

普通の物質の塊

(ダークハロー、銀河)

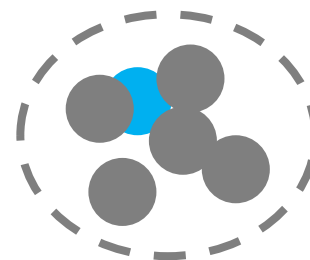
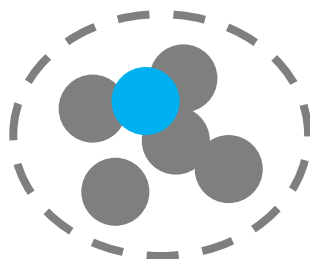
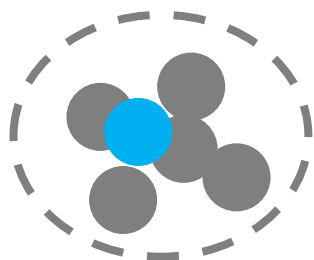


宇宙の構造形成



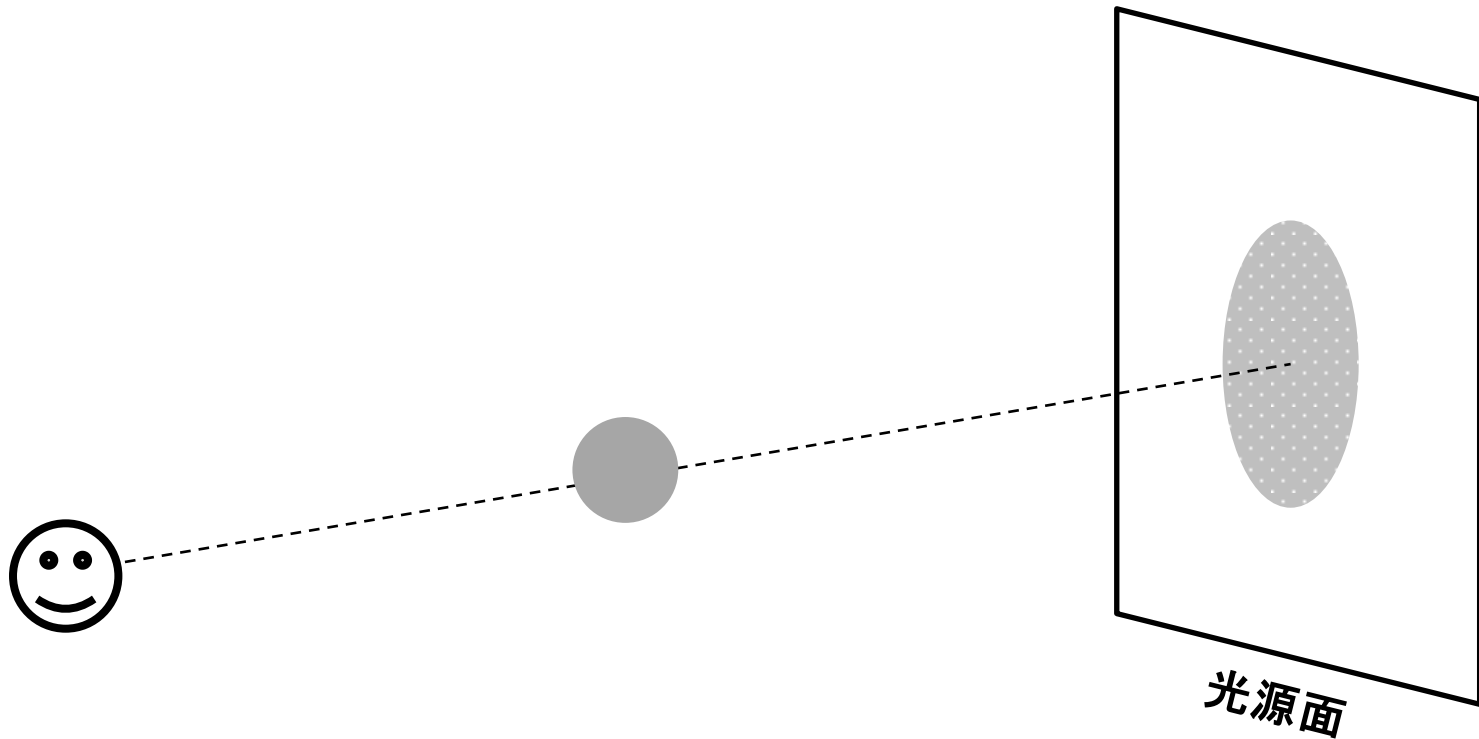
負質量の物質

普通の天体に混じって存在



遠方天体が手前のレンズ天体により多重像を作る確率

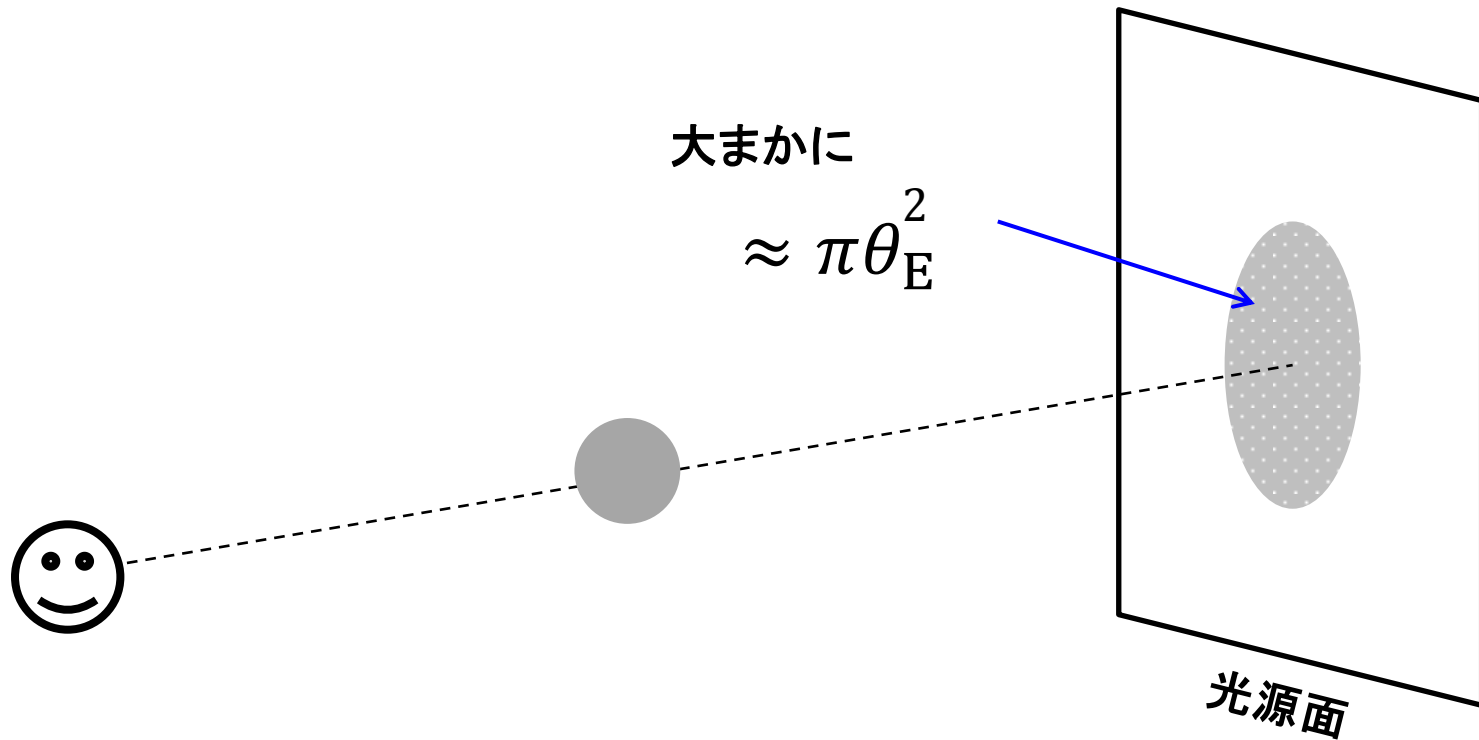
(Press & Gunn 1973; Turner, Ostriker & Gott 1984)



$$\text{確率} = \frac{\text{多重像として観測される(光源面上での)面積}}{\text{全天の立体角}(=4\pi)}$$

遠方天体が手前のレンズ天体により多重像を作る確率

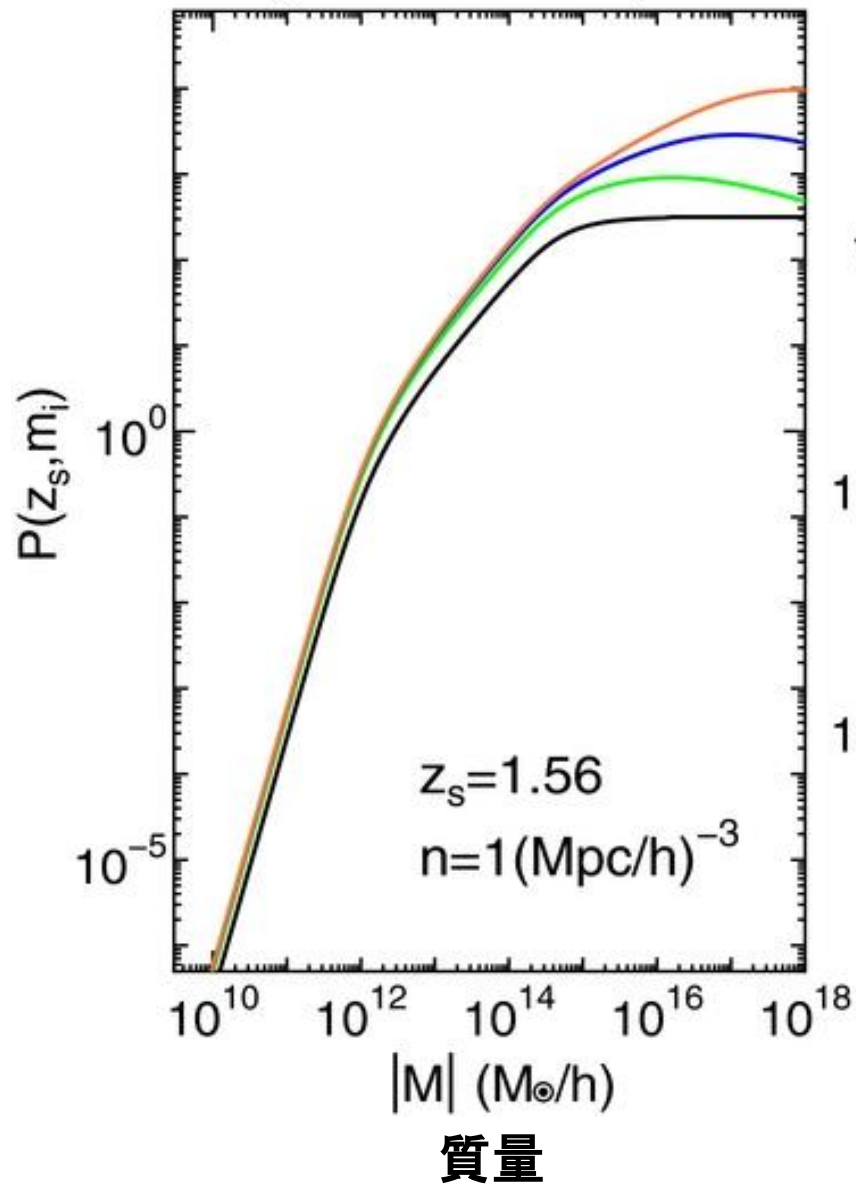
(Press & Gunn 1973; Turner, Ostriker & Gott 1984)



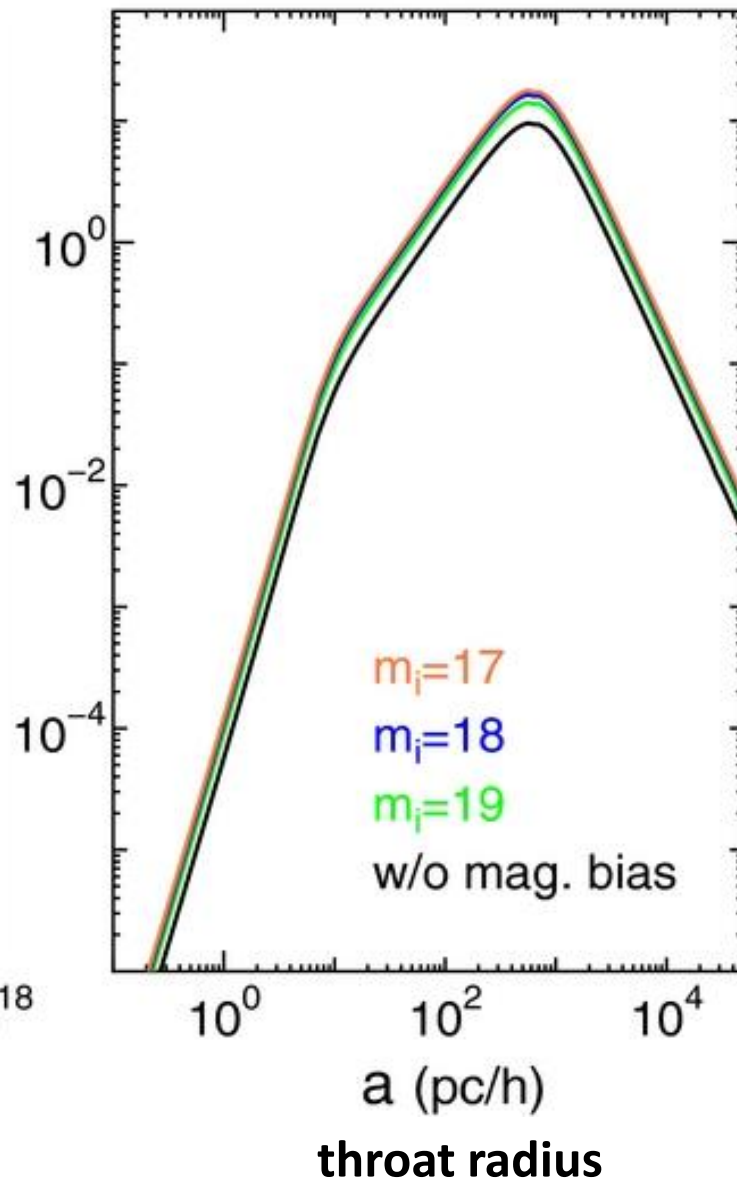
$$\text{確率} = \frac{\text{多重像として観測される(光源面上での)面積}}{\text{全天の立体角}(=4\pi)}$$

レンズ確率

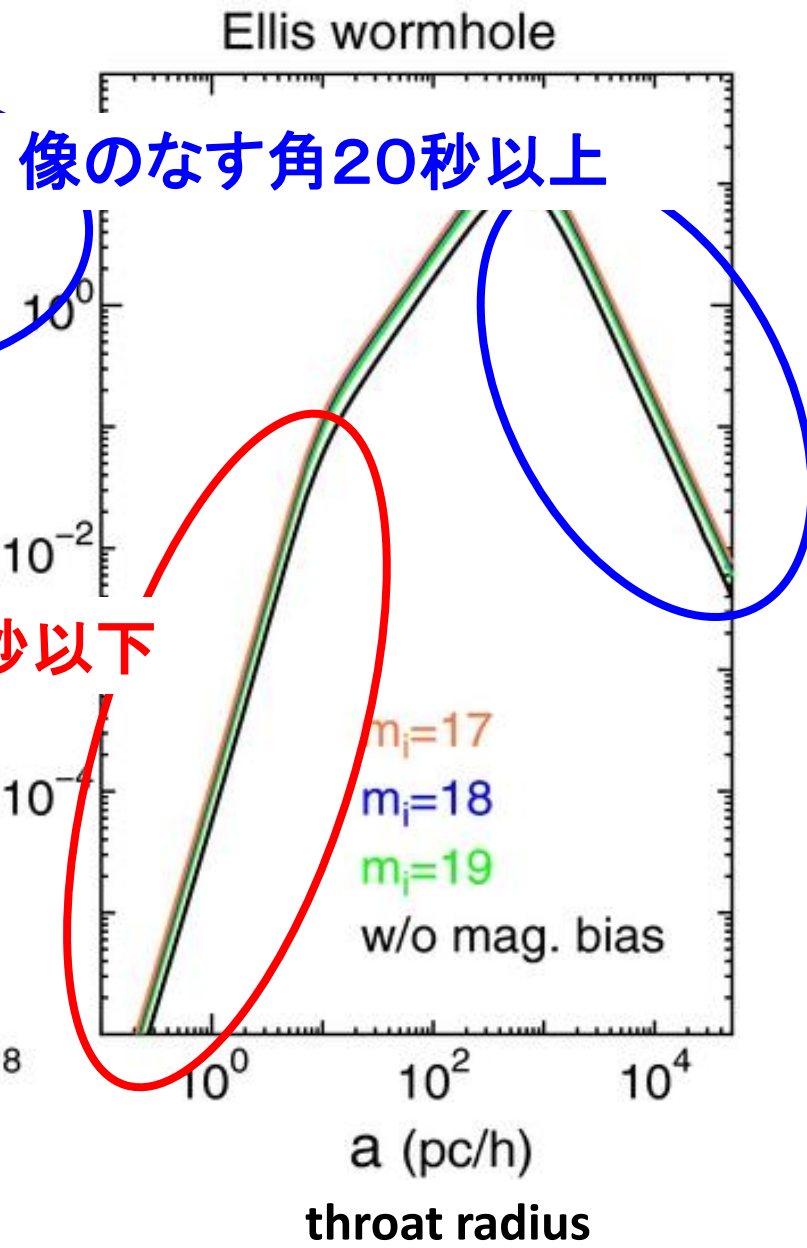
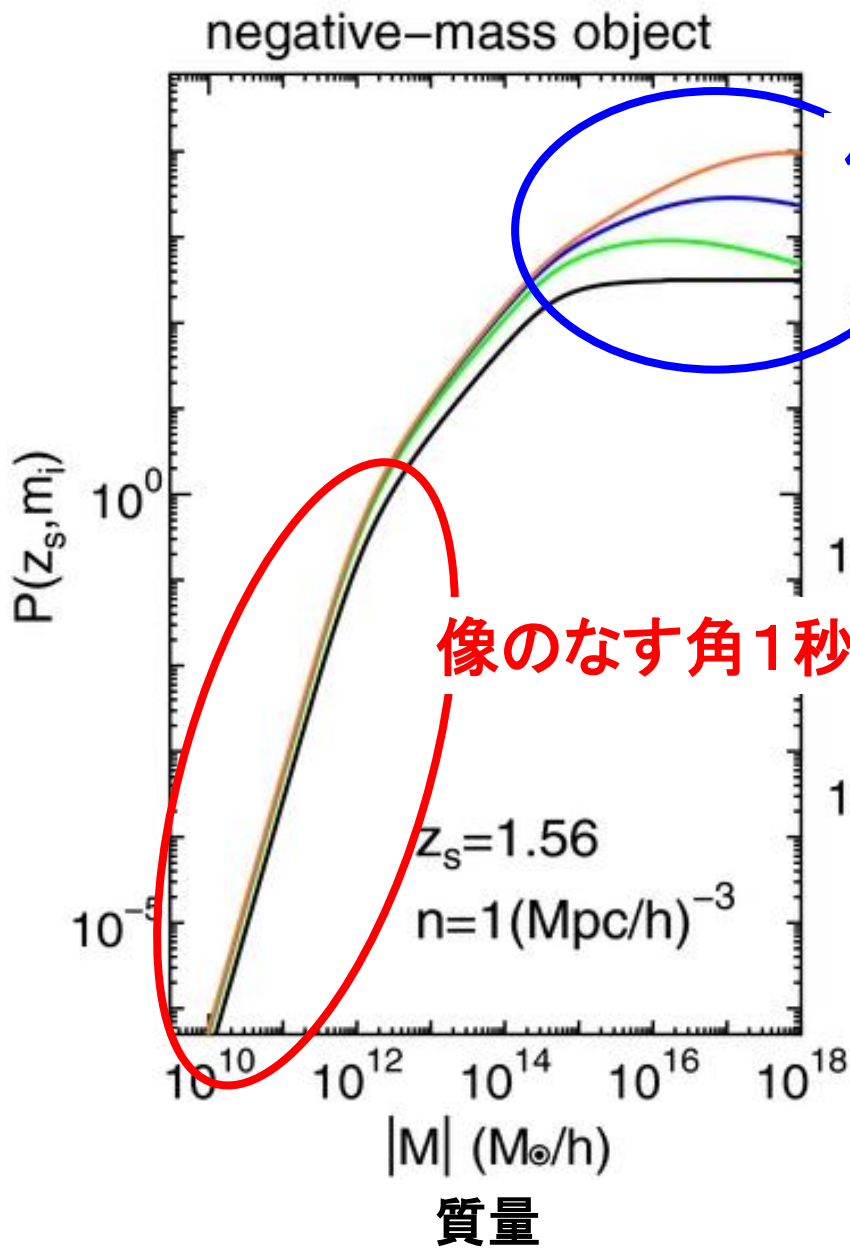
negative-mass object



Ellis wormhole



レンズ確率



像のなす角20秒以上

像のなす角1秒以下

Inertial and Gravitational mass (Jammer 1961,1999)

- Inertial mass

$$m_I$$

Eq. of motion

$$m_I a = F$$

- Gravitational mass

$$m_G$$

Gravitational force

$$F = G \frac{m_G M_G}{r^2}$$

Gravitational mass is Negative

(Inertial mass can be positive or negative)

Dynamics of Negative Masses under Newtonian mechanics

- Case of $m_G < 0$ & $m_I > 0$
 - Ordinary matter ($m_G > 0$ & $m_I > 0$)
 - N Negative mass ($m_G < 0$ & $m_I > 0$)

Two Negative masses



“attractive”

Dynamics of Negative Masses under Newtonian mechanics

- Case of $m_G < 0$ & $m_I < 0$
- Ordinary matter ($m_G > 0$ & $m_I > 0$)
- N Negative mass ($m_G < 0$ & $m_I < 0$)

Ordinary matter and Negative mass



“Ordinary Matter Escapes from Negative Mass”

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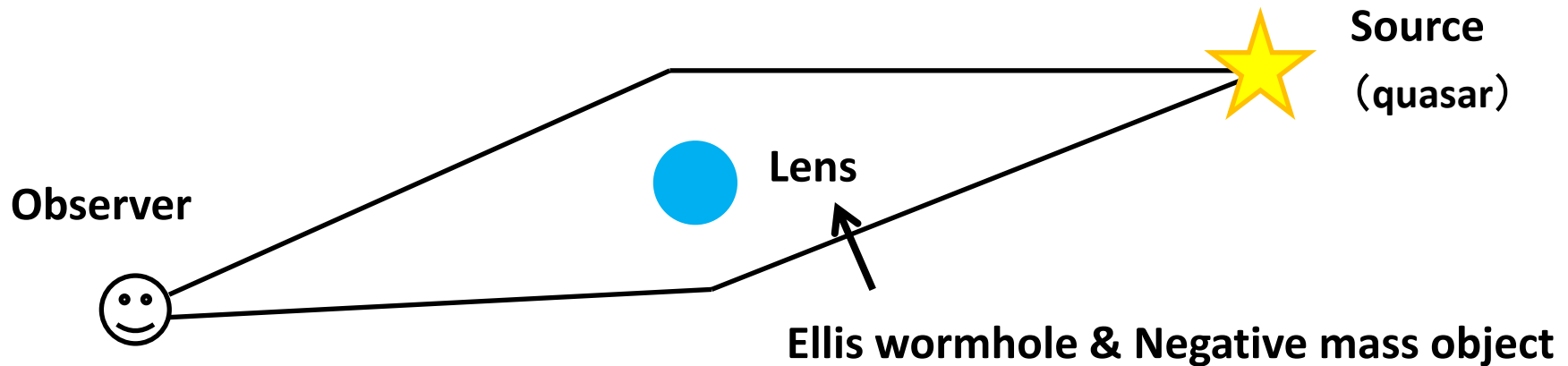
Two Negative masses



“Repulsive”

0. Abstract

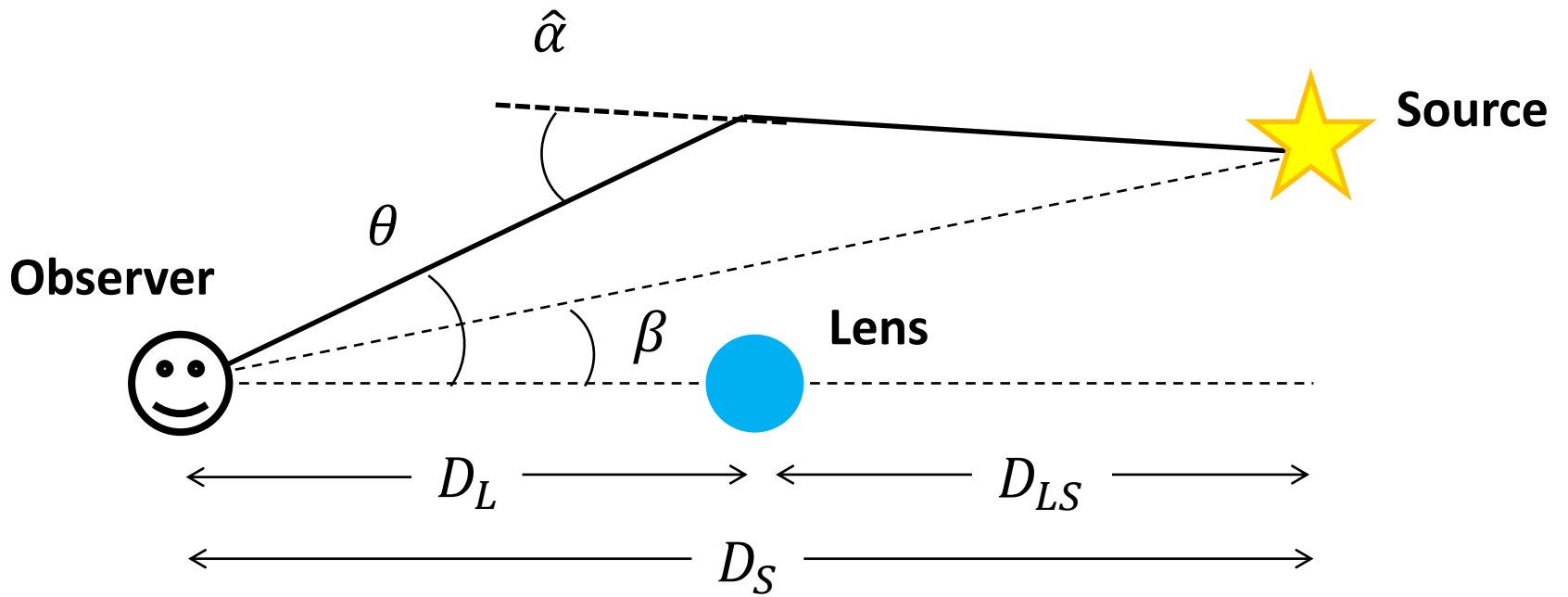
**observational constraint on cosmic abundances of
Ellis wormhole & Negative mass compact object
from SDSS quasar lens survey**



If there are Ellis wormholes and Negative mass objects in the Universe

- ➔ **Distant quasar seen as multiple images by gravitational lensing**
- ➔ **SDSS quasar survey didn't find such multiple images**
- ➔ **As a result, we can set an observational upper bound**

Lens Equation



Lens equation

$$\beta = \theta + \frac{\theta_E^2}{\theta}$$

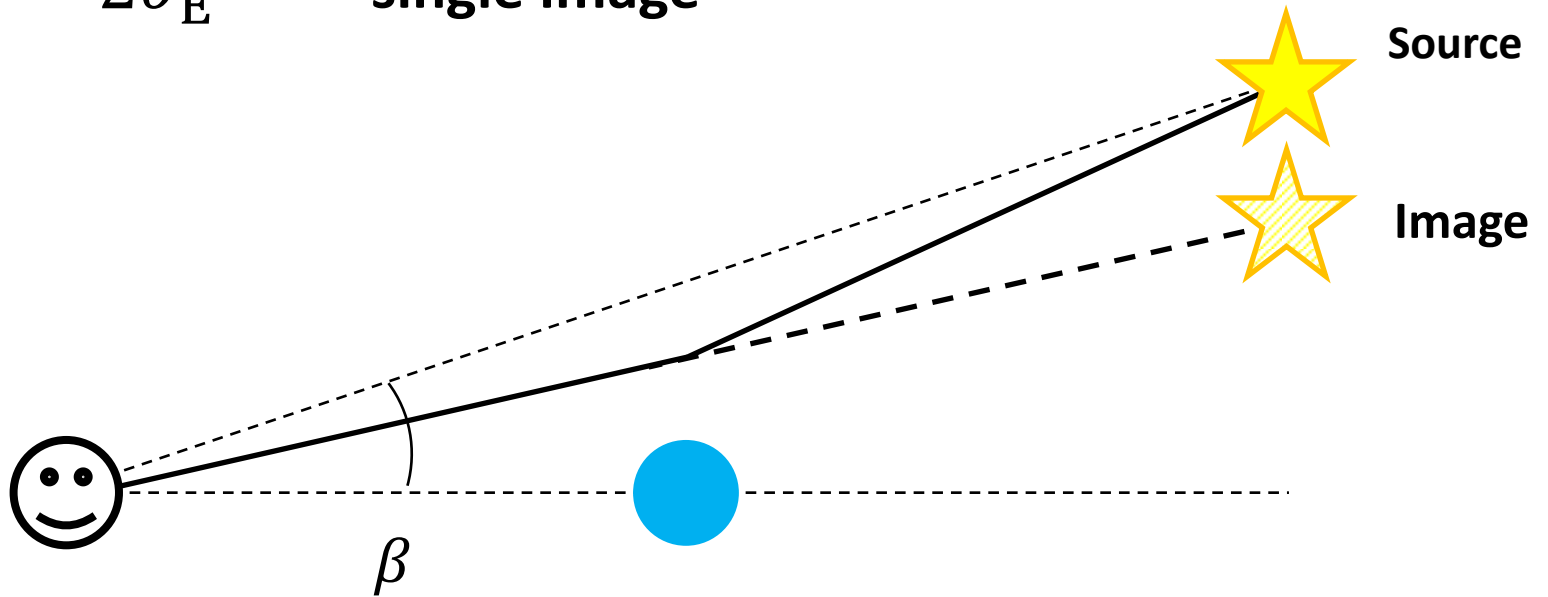
Einstein radius

$$\theta_E = \sqrt{4|M| \frac{D_{LS}}{D_L D_S}}$$

of images

$$\beta = 2\theta_E$$

single image



Infinite magnification